Master in Electrical and Electronics Engineering

EE-518: Analog Circuits for Biochip

Lecture #2

Equivalent Circuits for Bio/CMOS interfaces

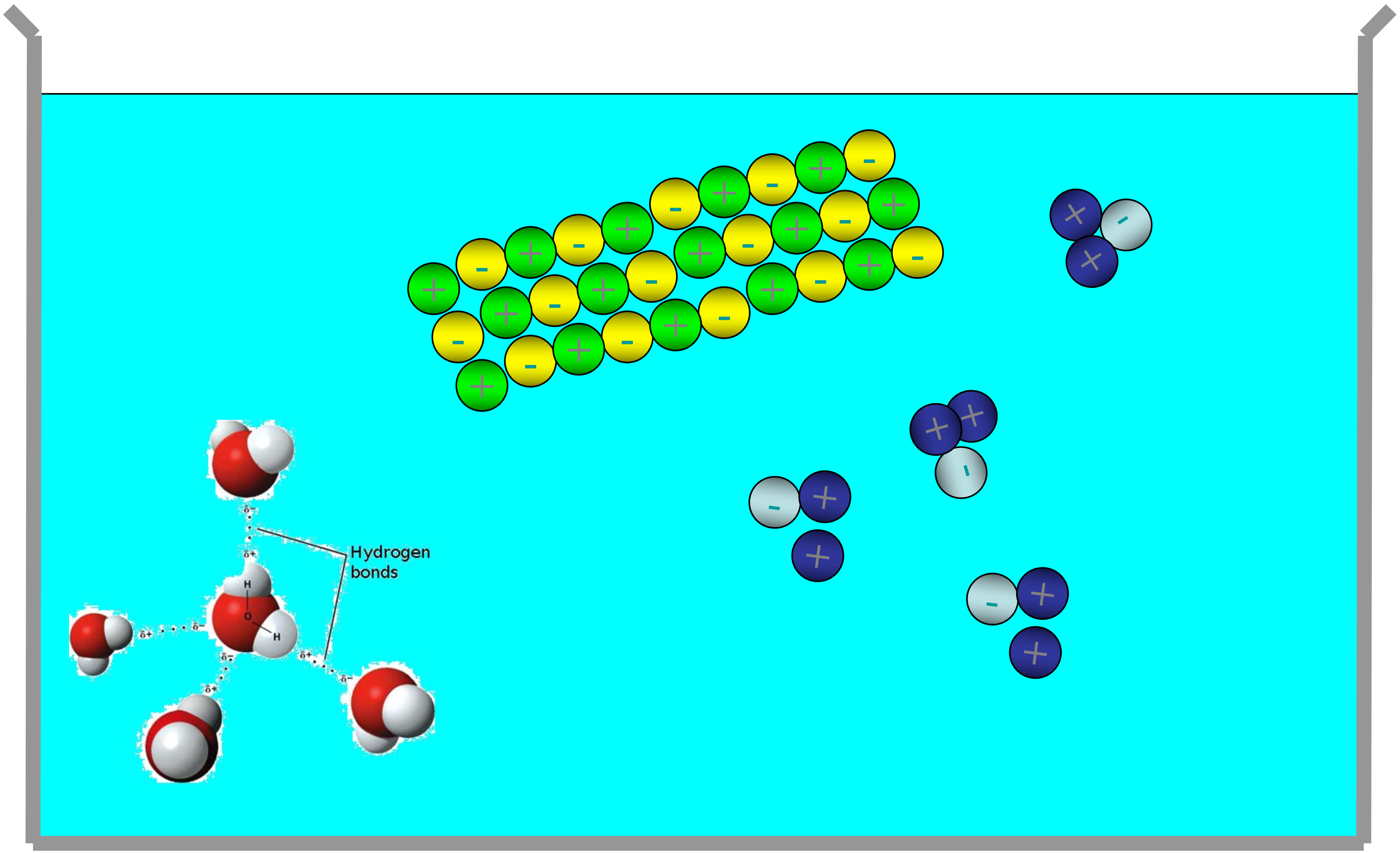
EPFL

Lecture Outline

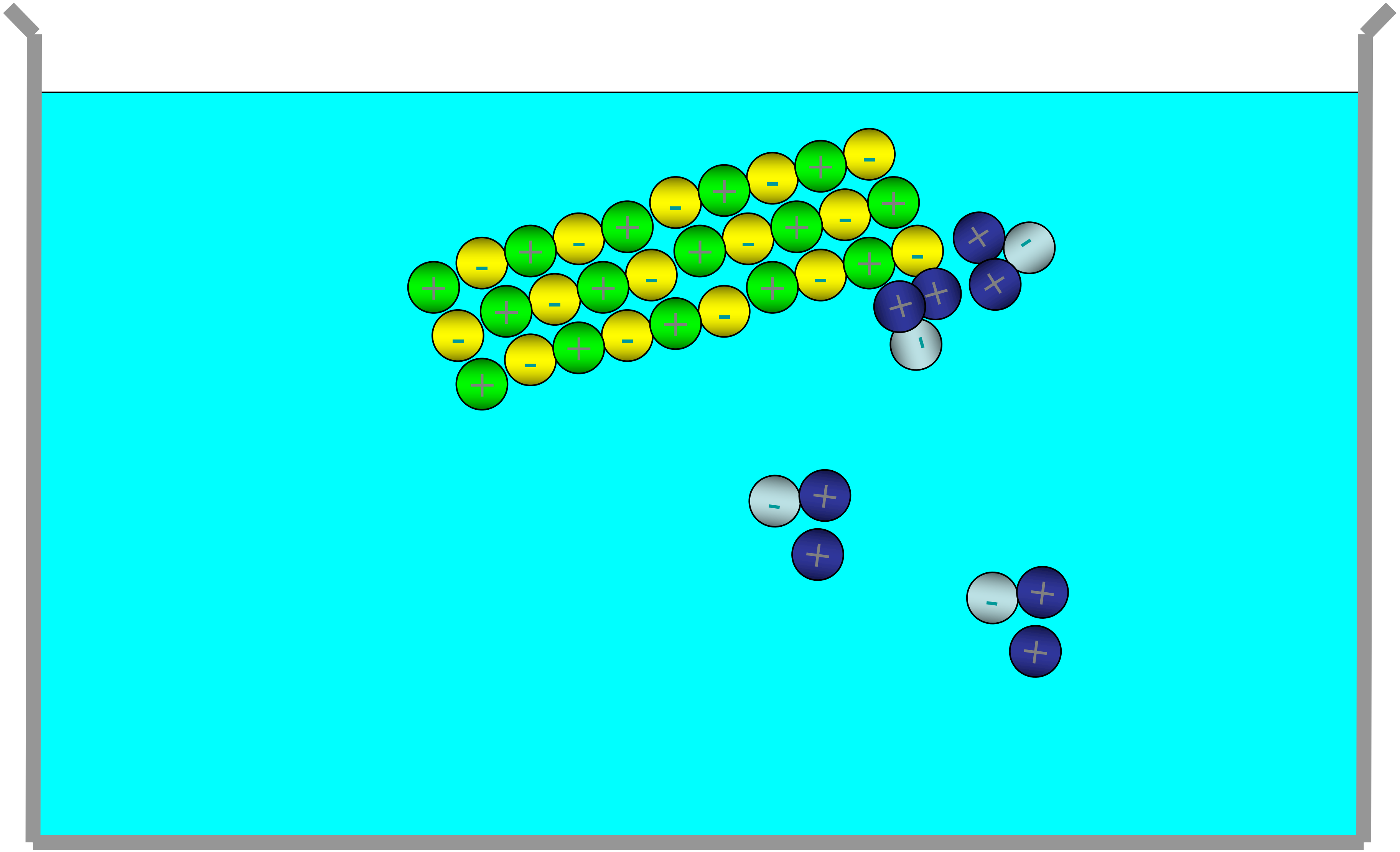
(Book Bio/CMOS (*2nd Ed.*): Chapter' paragraphs § 2.8-9; 5.4; 10.2)

- Passive model (Randles) for electrolyte solutions
- Passive model with CPE
- Passive model for Brain Interface
- Active model with Current Generator
- Active model with Voltage Effects

Liquid Solution: Ionic Solid in liquid



Liquid Solution: Ionic Solid in liquid



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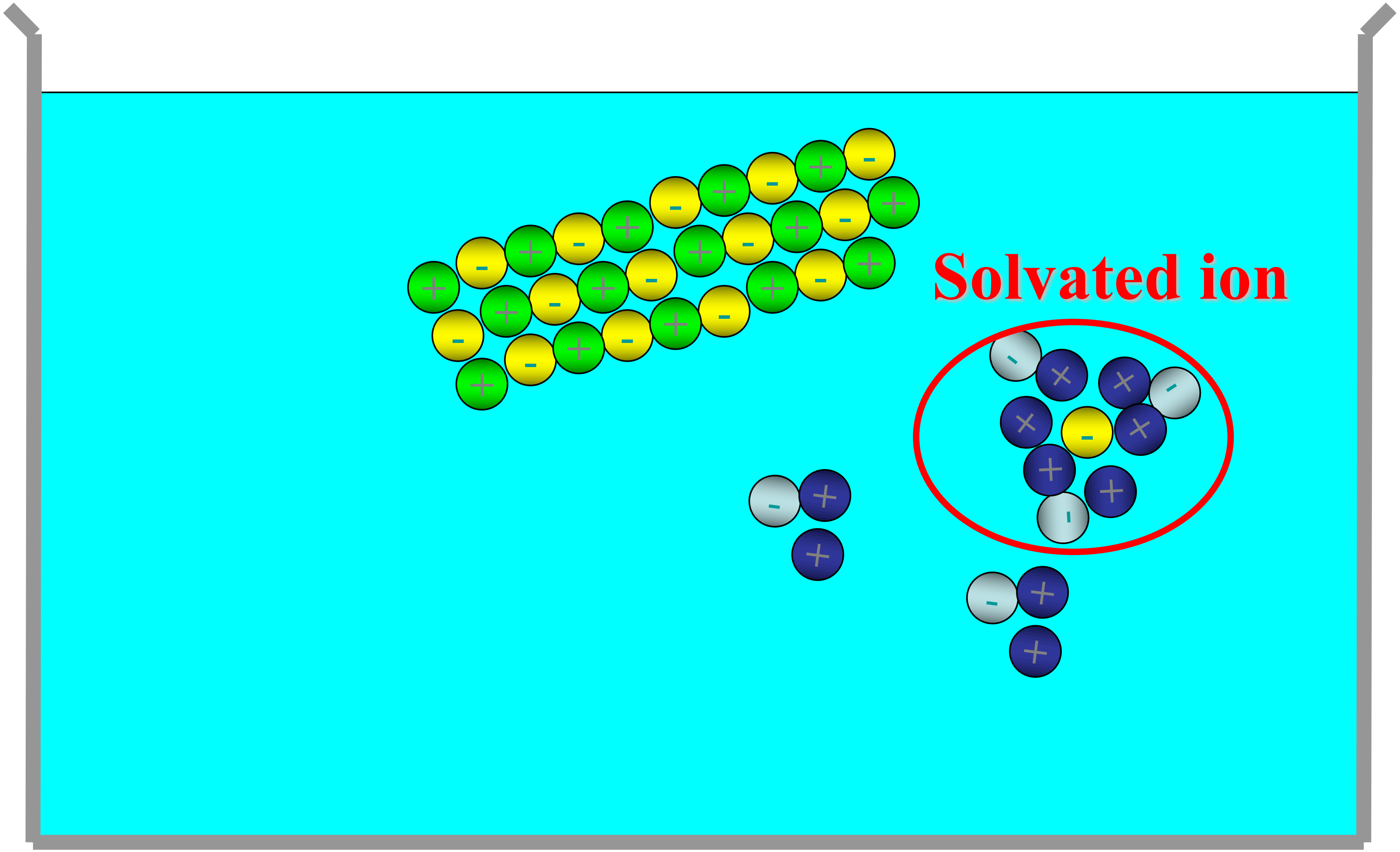
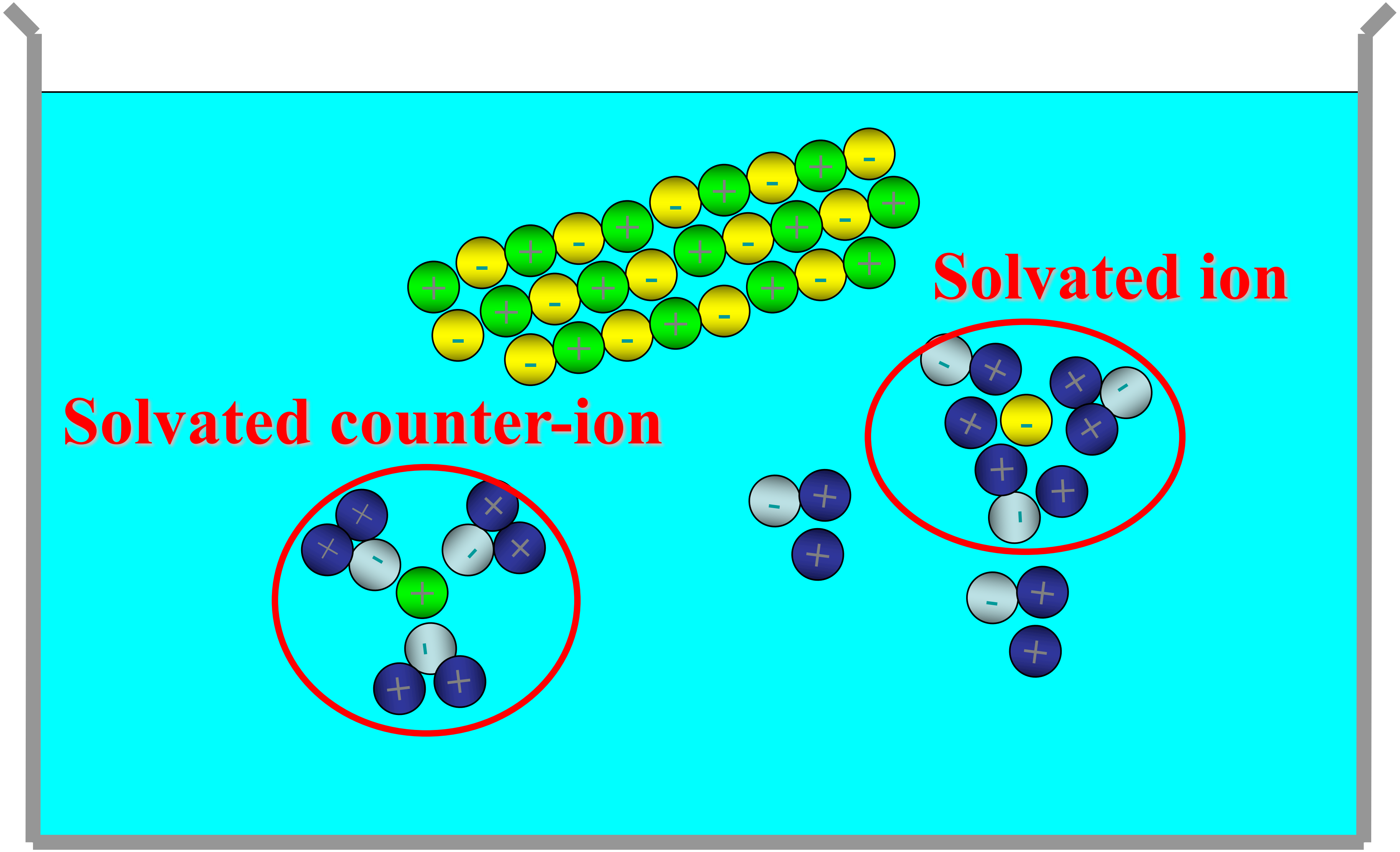
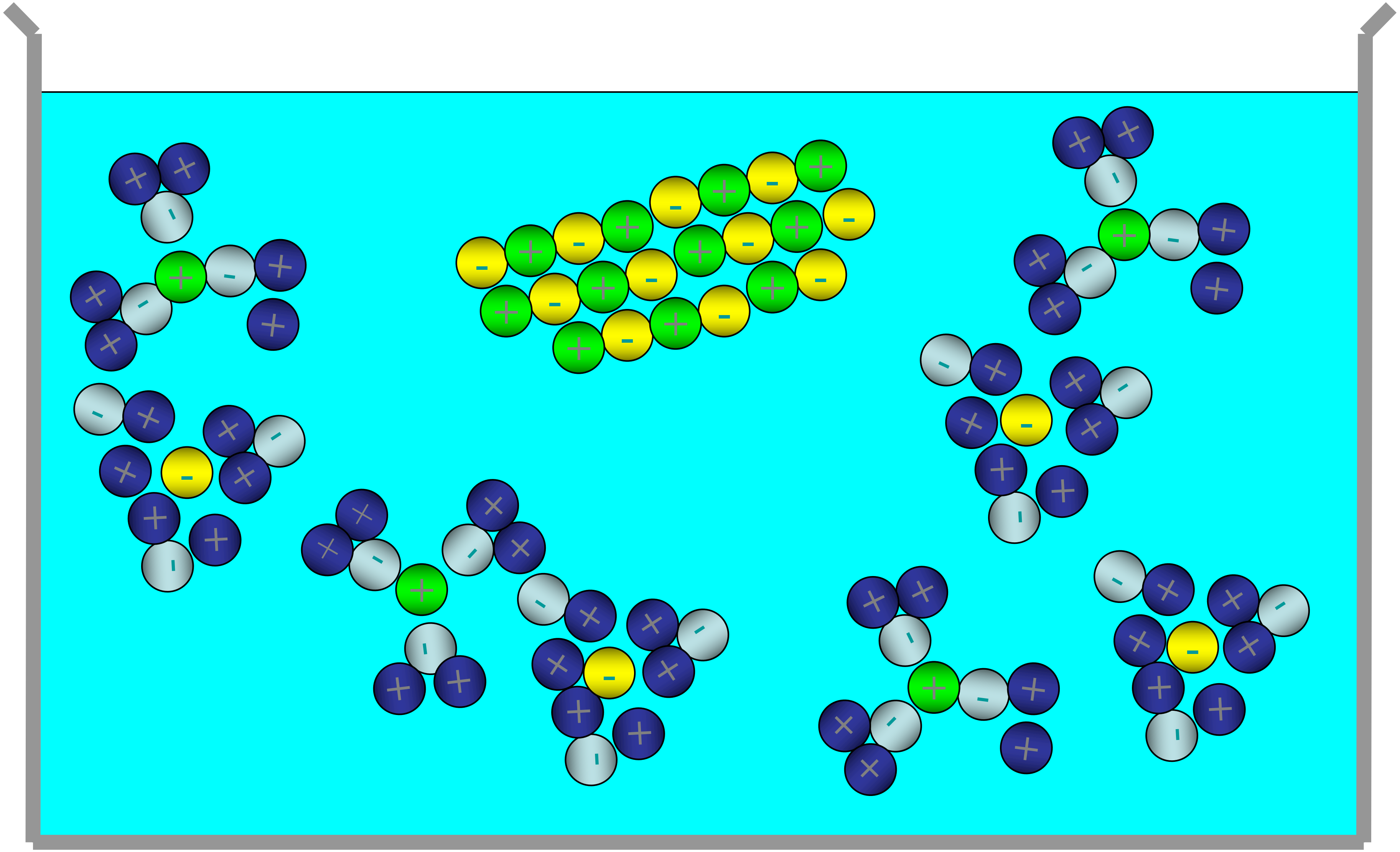


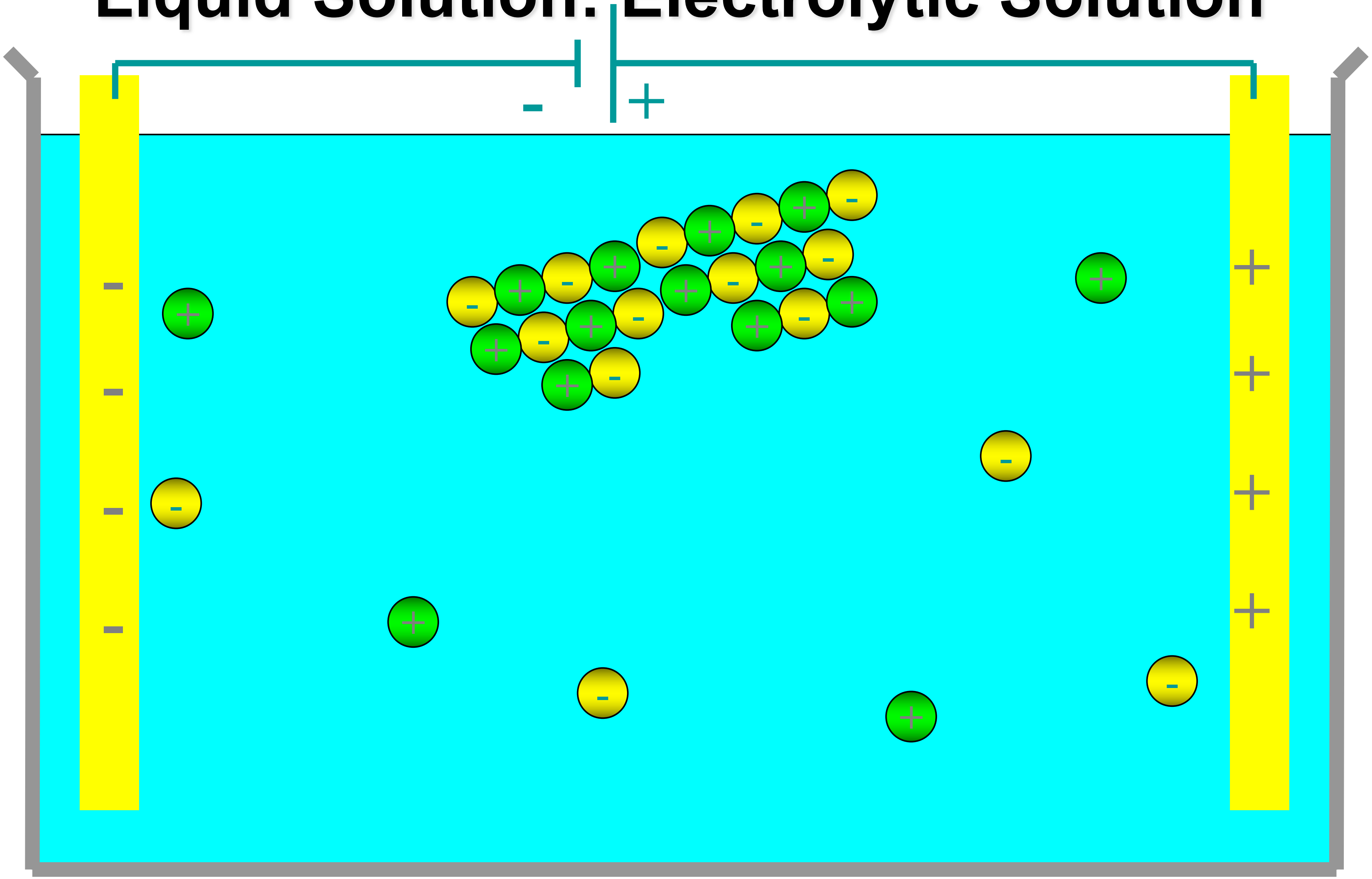
Liquid Solution: Ionic Solid in liquid



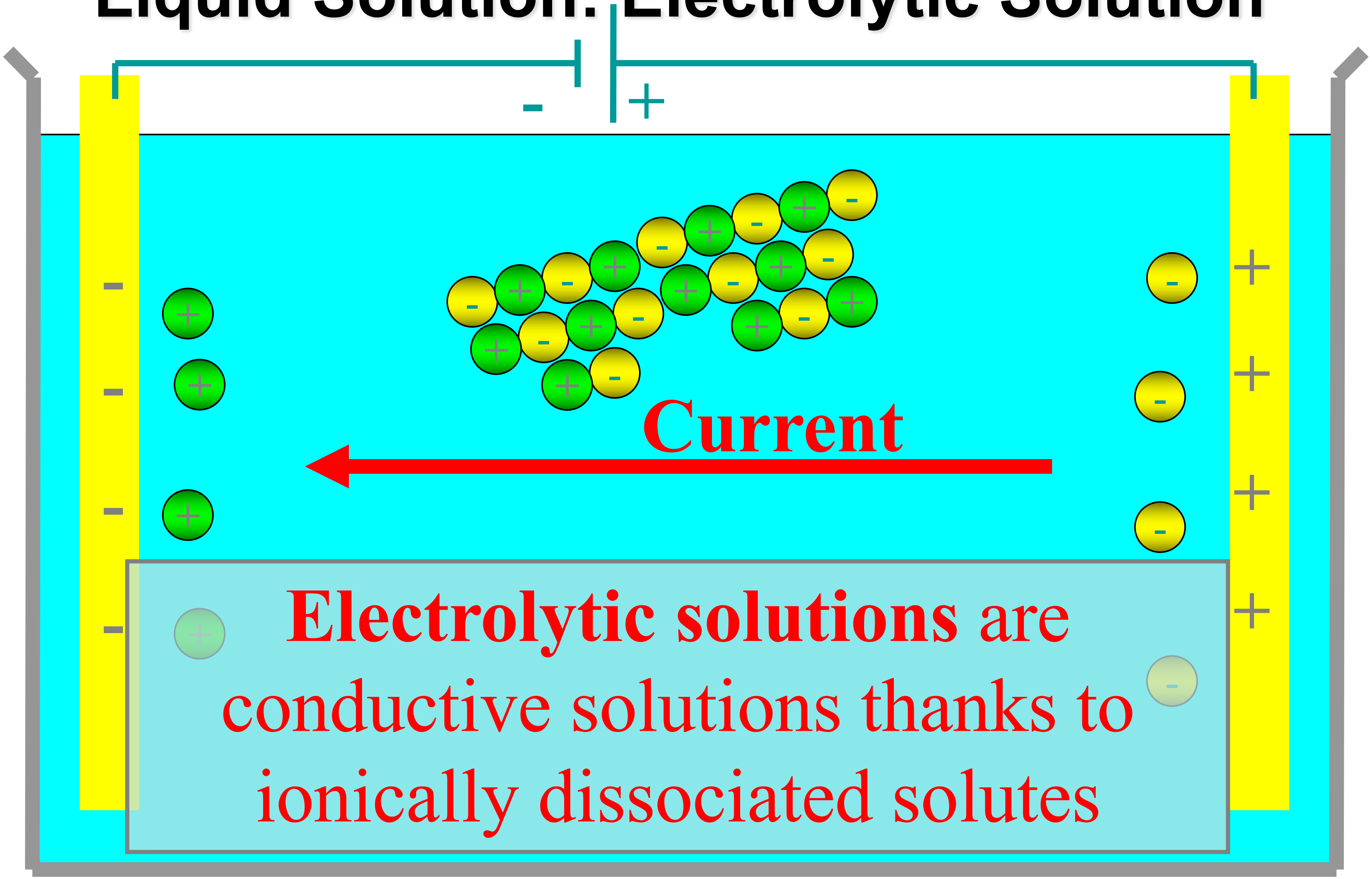
Liquid Solution: Ionic Solid in liquid



Liquid Solution: Electrolytic Solution



Liquid Solution: Electrolytic Solution



Electrolytic solutions are conductive solutions thanks to ionically dissociated solutes

Ionic Solution of HCl

Dissociation of Chloride Acid

$$2HCl + 2H_2O \rightarrow 2H_3O^+ + 2Cl^-$$

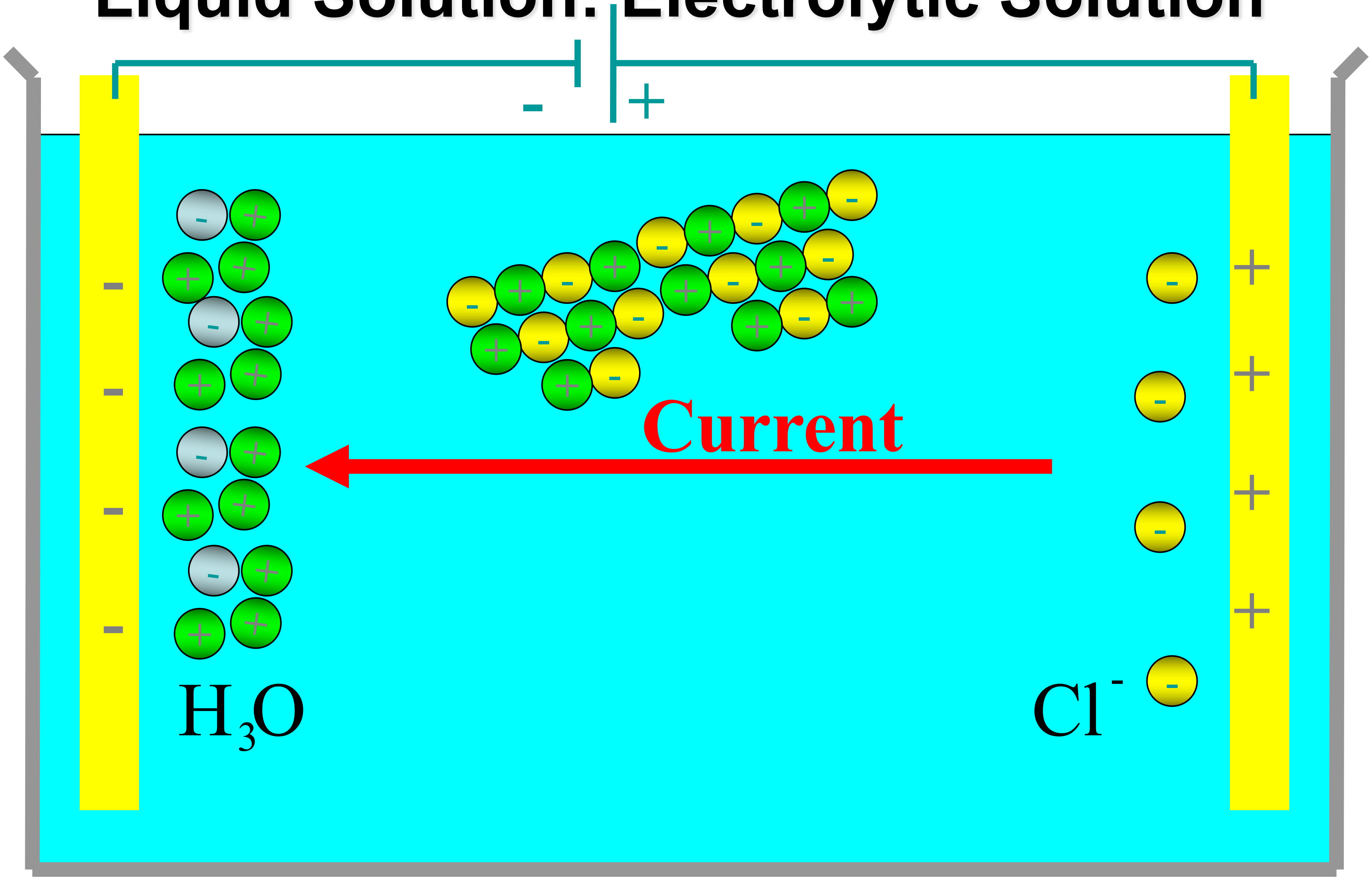
$$2H_3O^+ + 2e^- \rightarrow 2H_2O + H_2 \uparrow$$

– Electrode (Cathode)

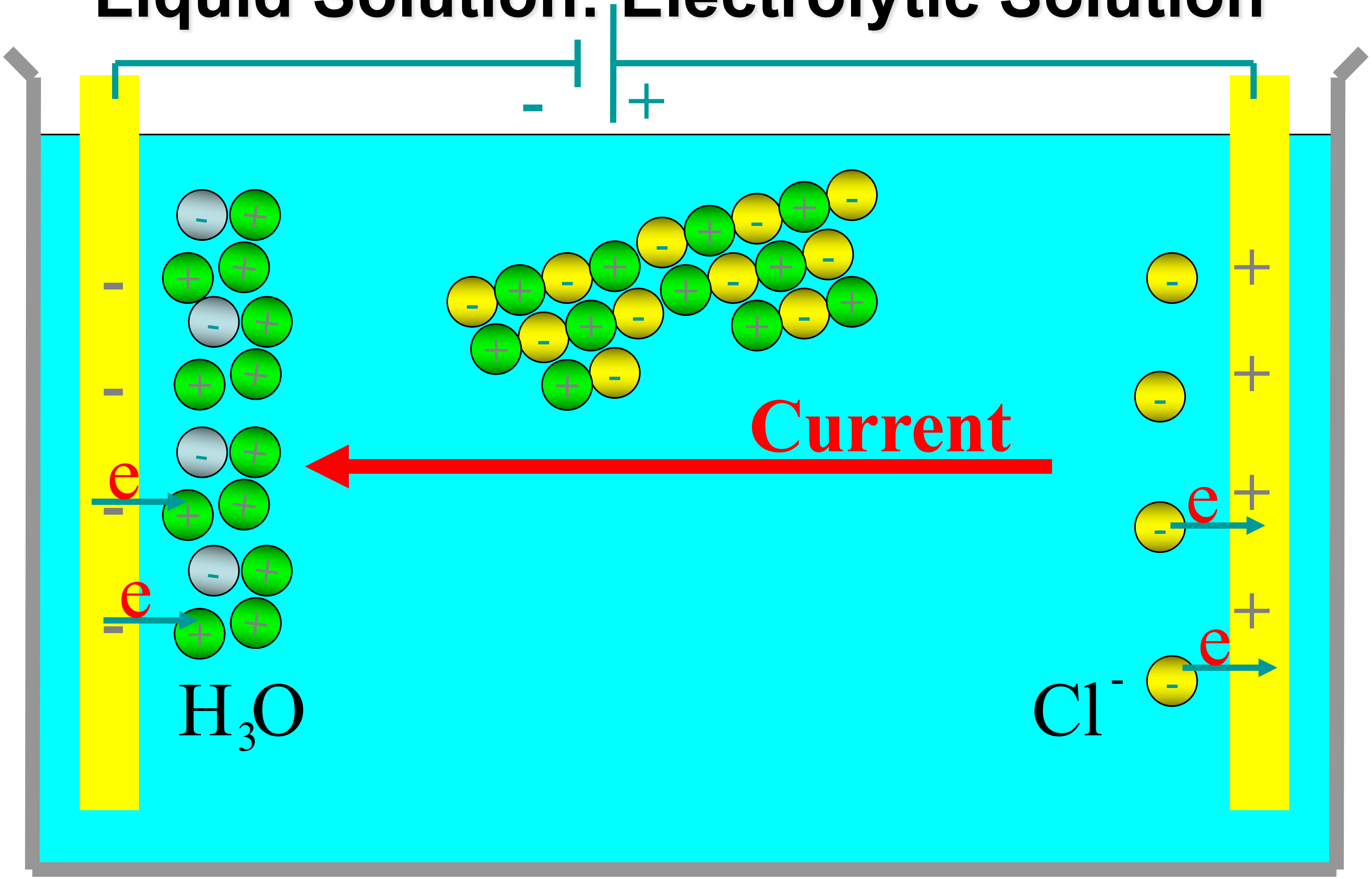
$$2Cl^- \rightarrow Cl_2 \uparrow + 2e^-$$

+ Electrode (Anode)

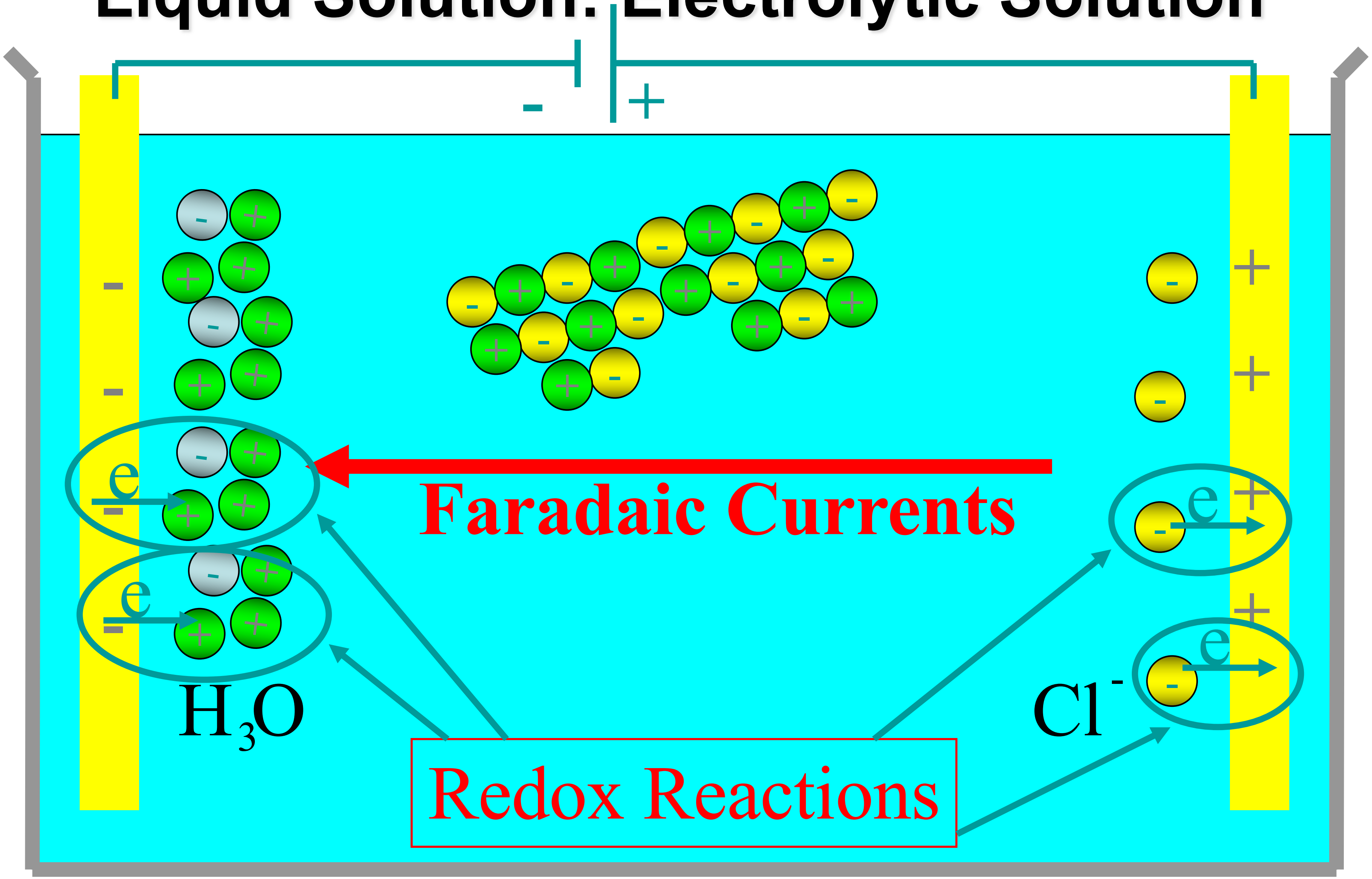
Liquid Solution: Electrolytic Solution



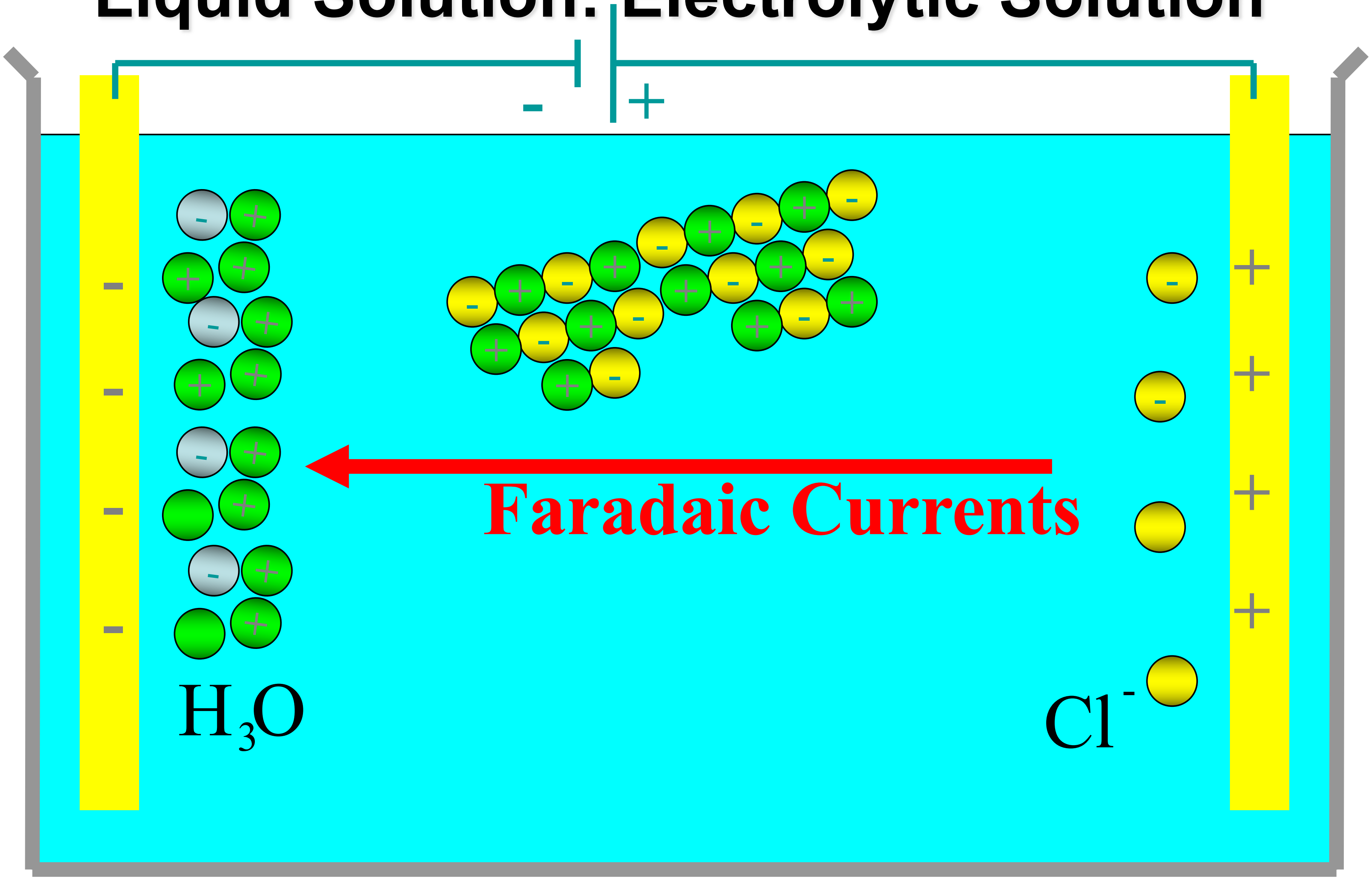
Liquid Solution: Electrolytic Solution



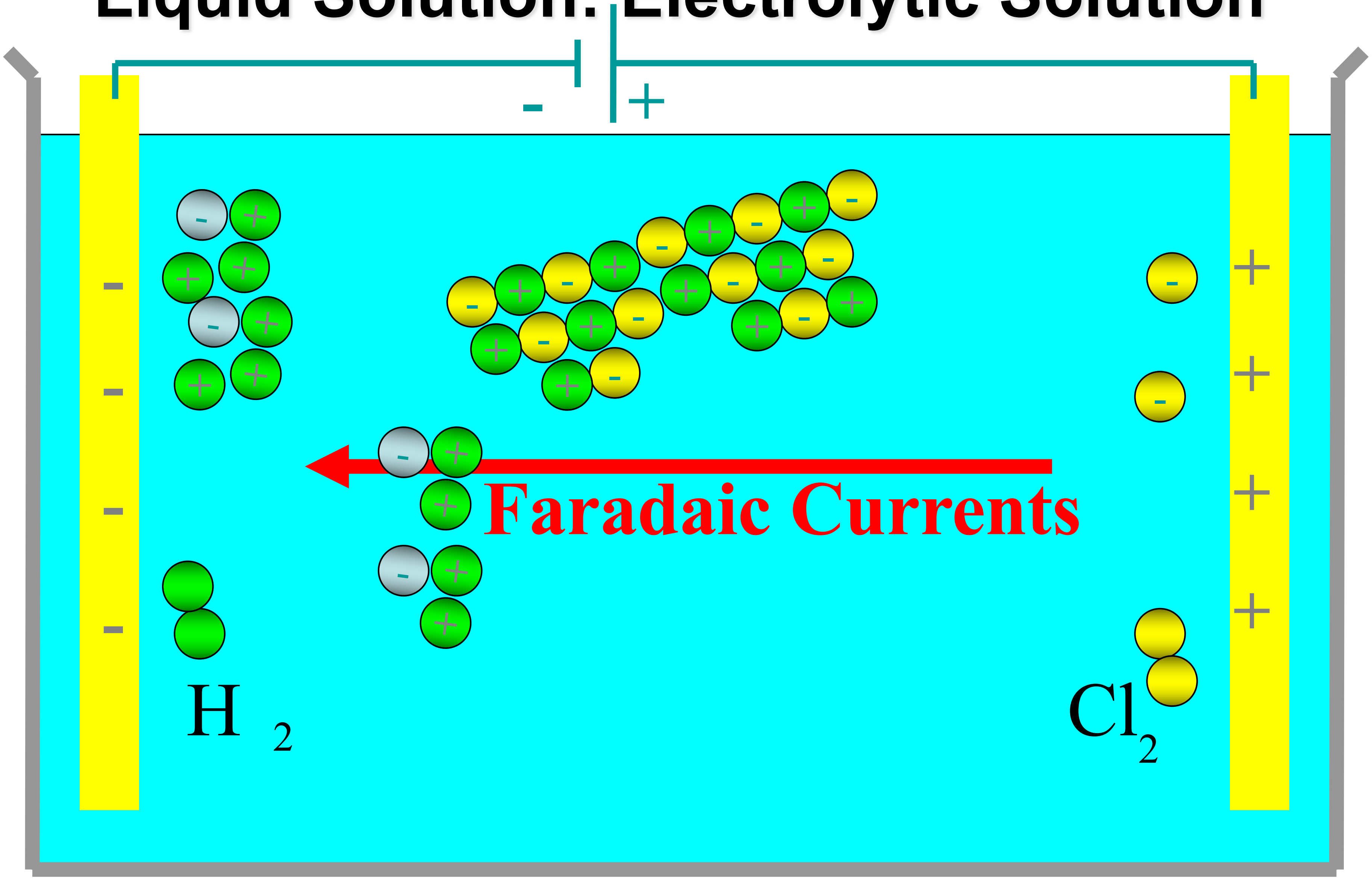
Liquid Solution: Electrolytic Solution



Liquid Solution: Electrolytic Solution



Liquid Solution: Electrolytic Solution



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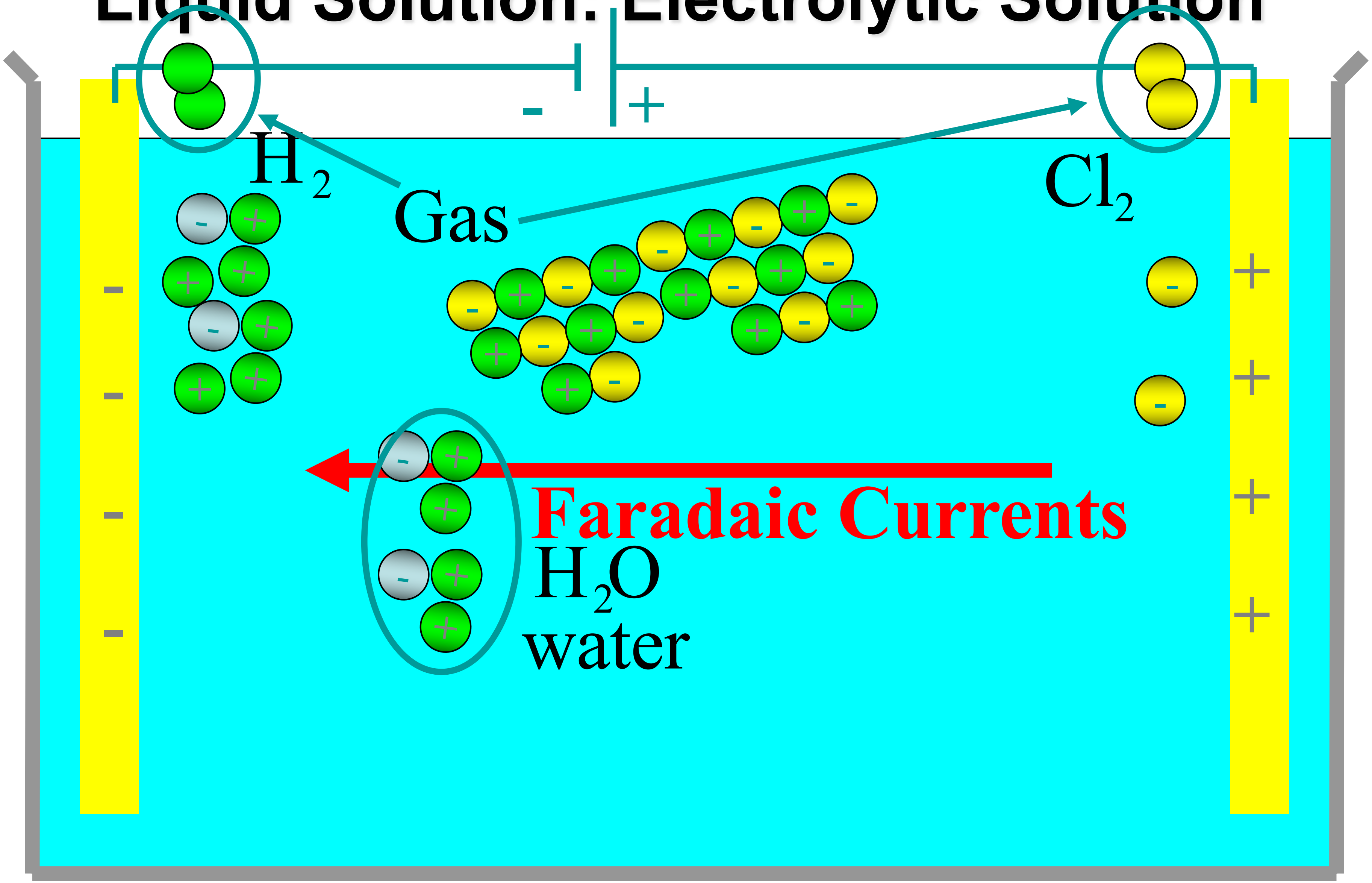
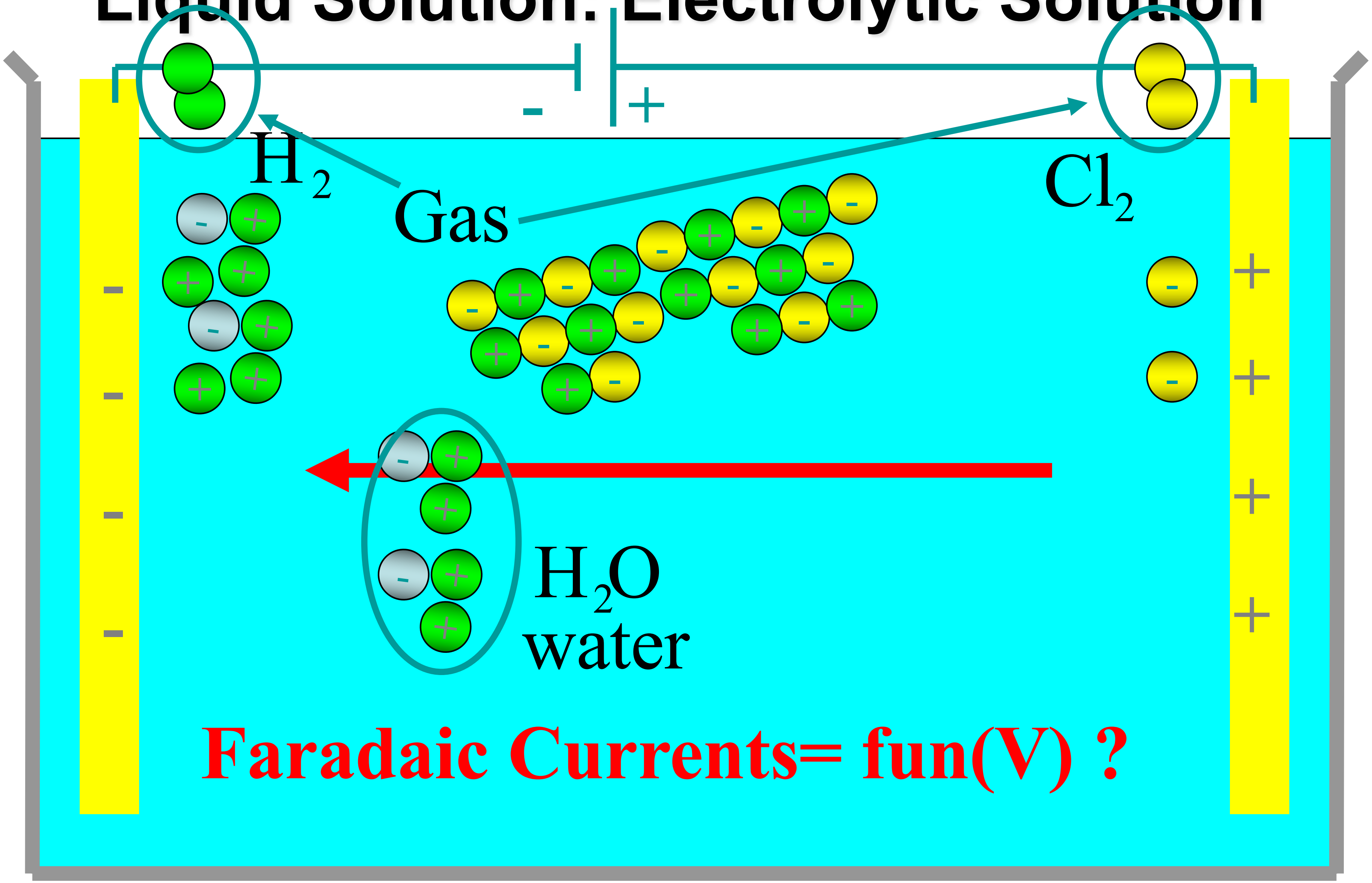
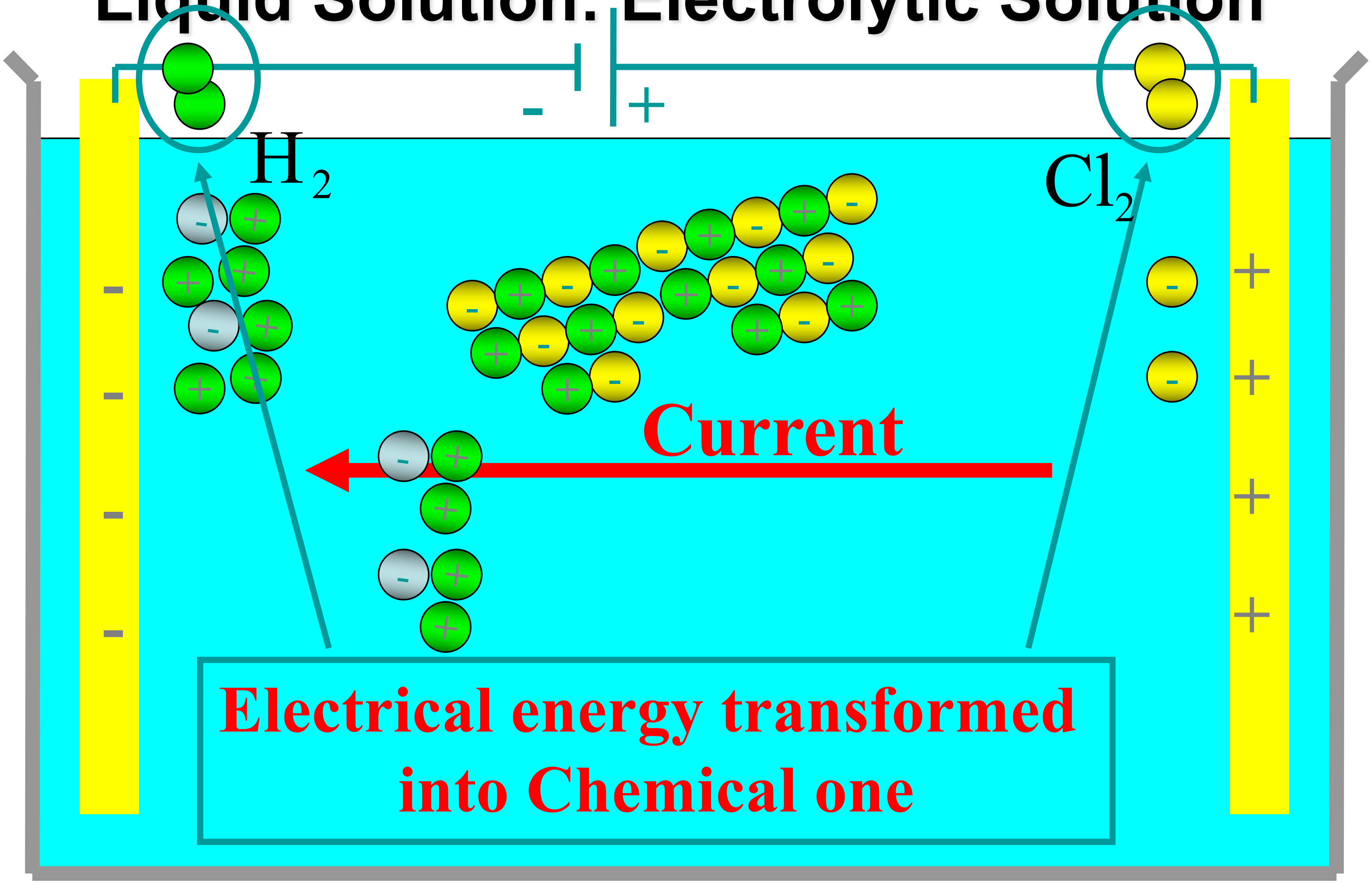


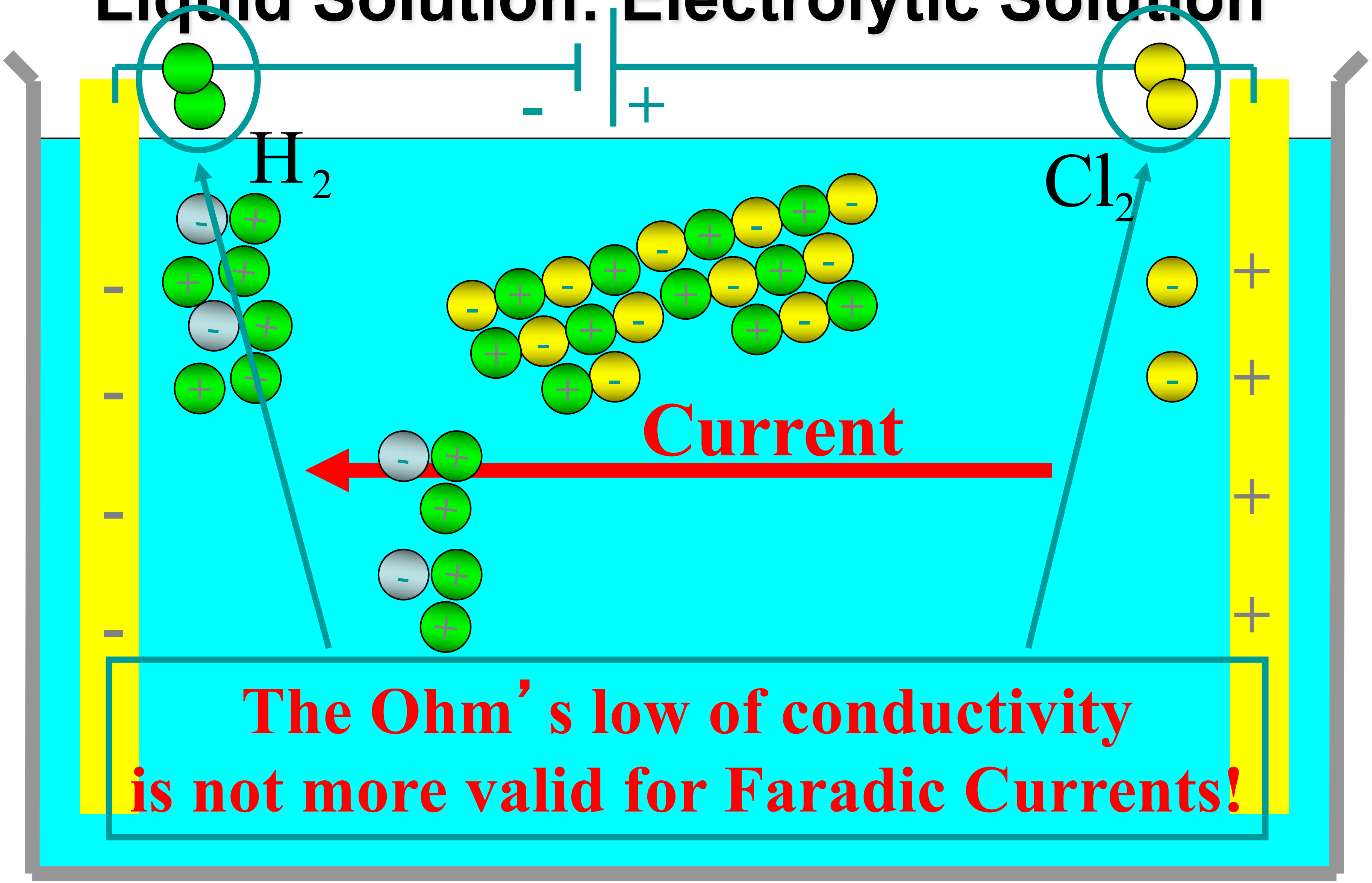
Liquid Solution: Electrolytic Solution



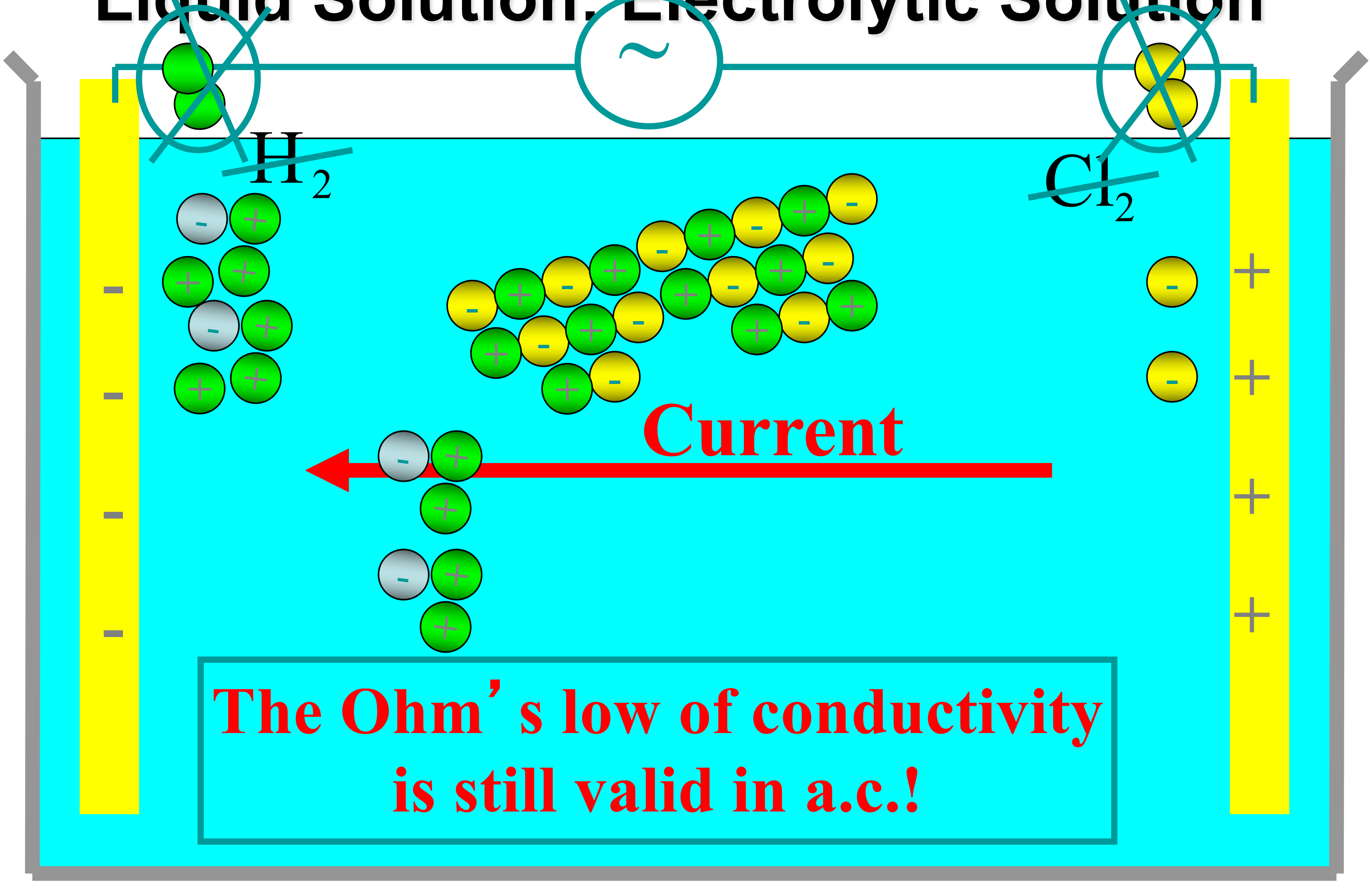
Liquid Solution: Electrolytic Solution



Liquid Solution: Electrolytic Solution

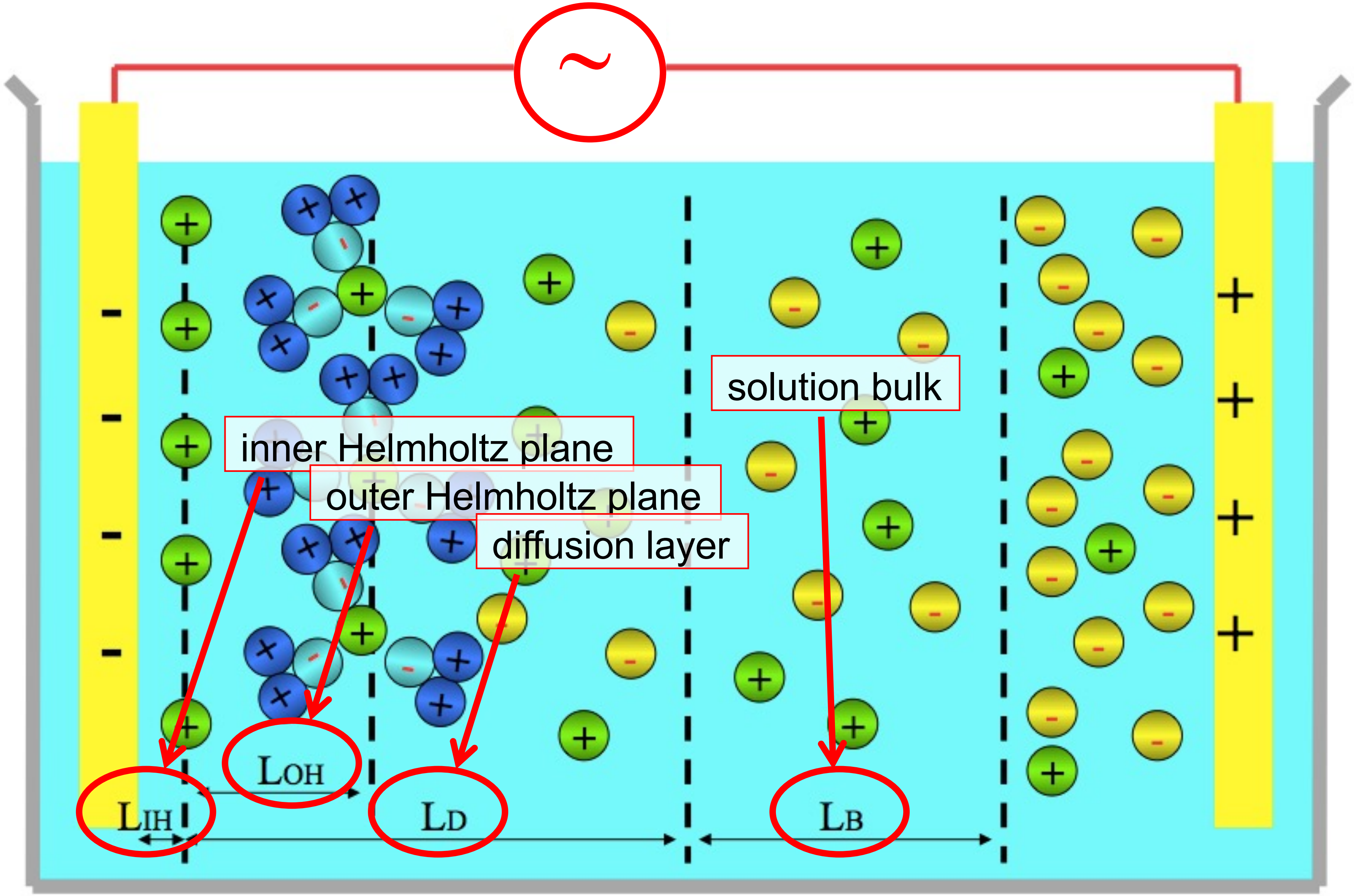


Liquid Solution: Electrolytic Solution

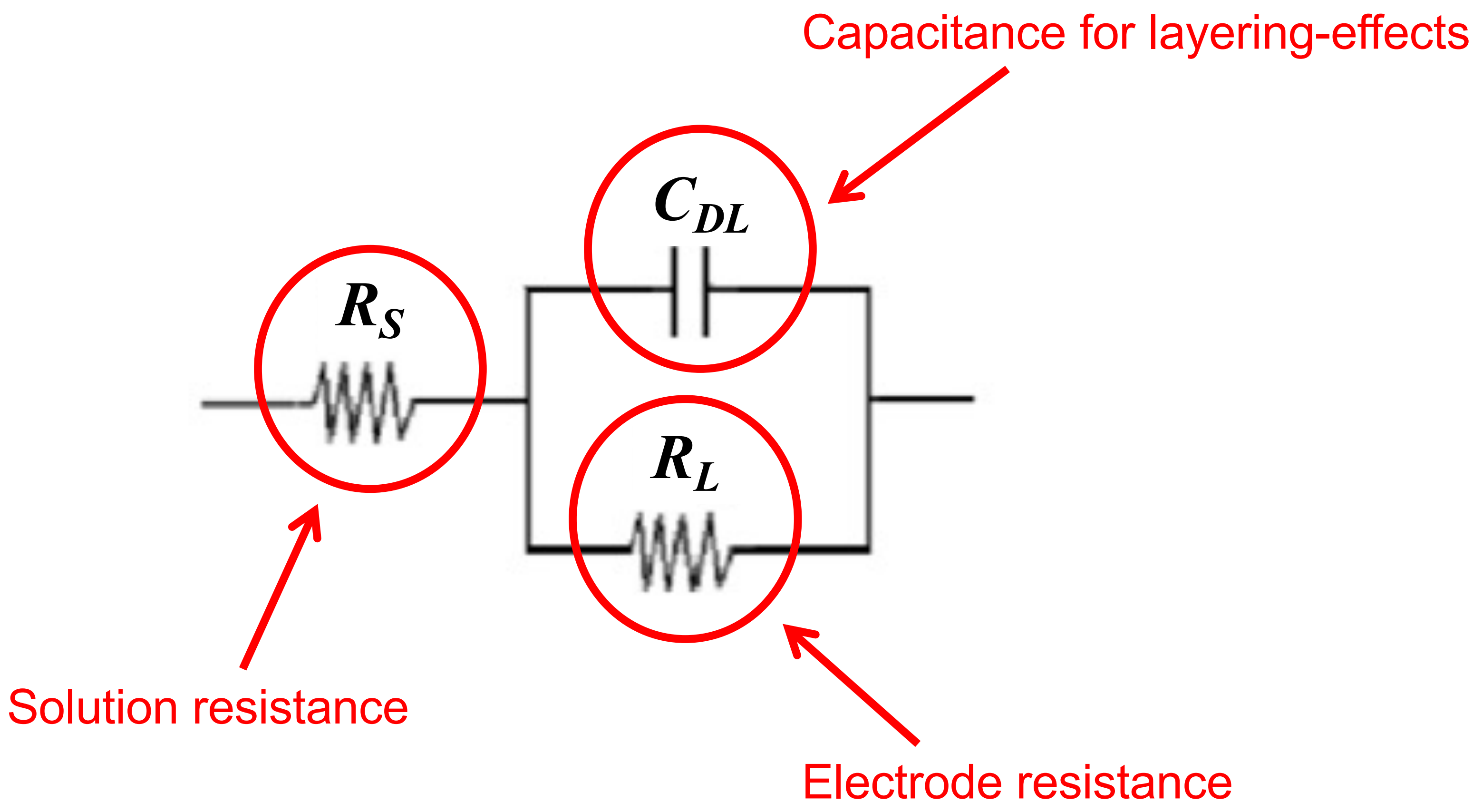


The Ohm's low of conductivity is still valid in a.c.!

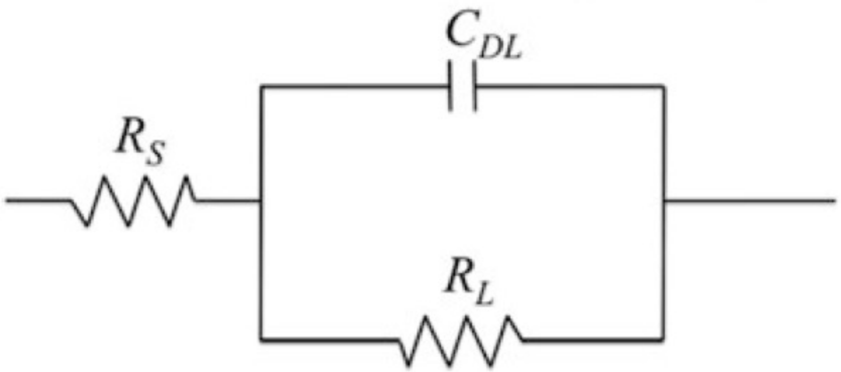
Helmholtz Planes



Equivalent Circuit with Layering effects: Randles Model



Randle Model



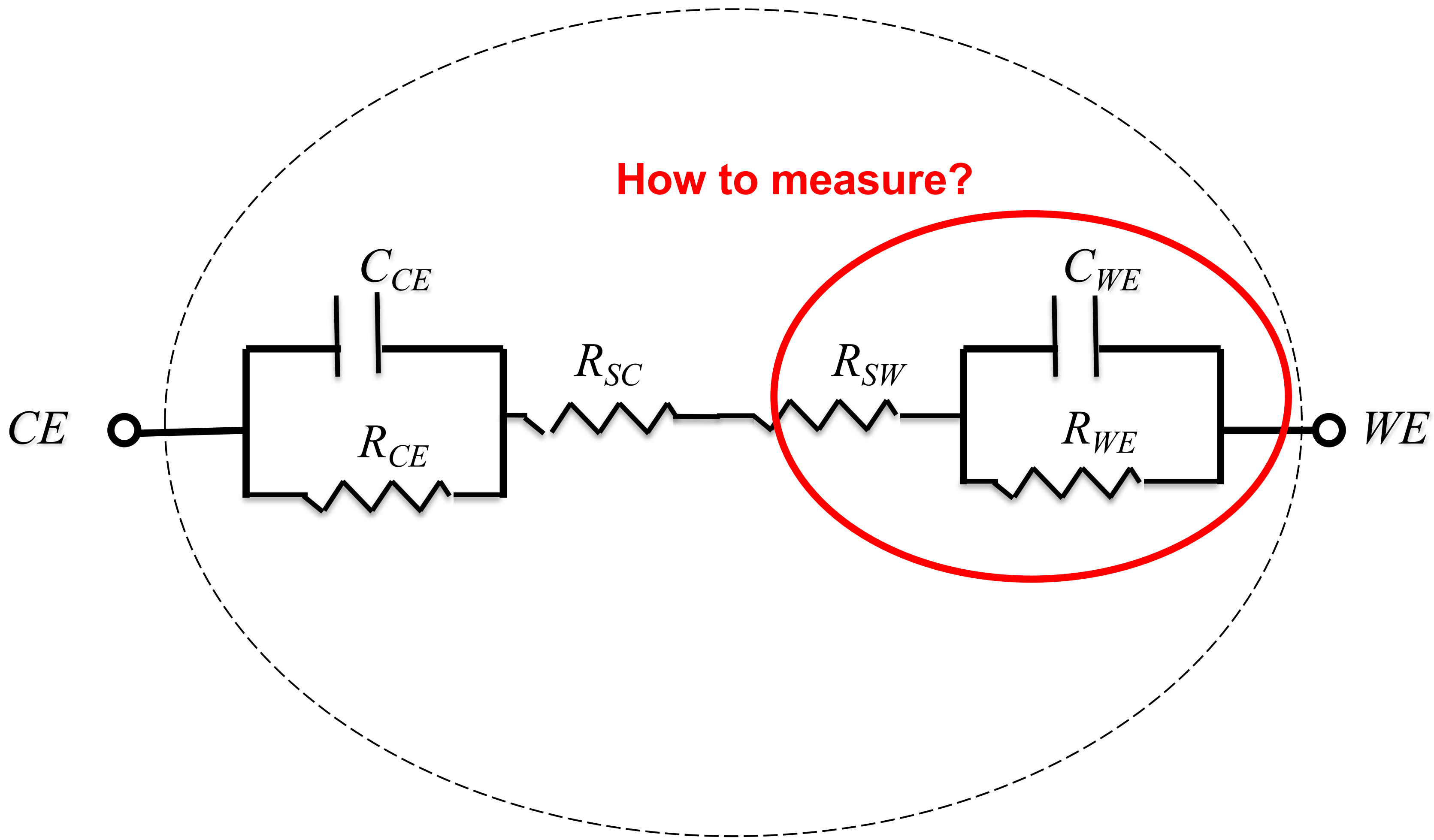
$$C_{DL} = \varepsilon_{DNA}\varepsilon_o \frac{A}{d}$$

$$Z_{//} = \frac{R_L}{j\omega C_{DL} R_L + 1}$$

$$I_{non-F} = \frac{V_{ref}}{Z} = \frac{1 + j\omega C_{DL} R_L}{R_L} V_{ref}$$

Equivalent circuits of DNA Bio/CMOS interface

Equivalent circuit: passive model



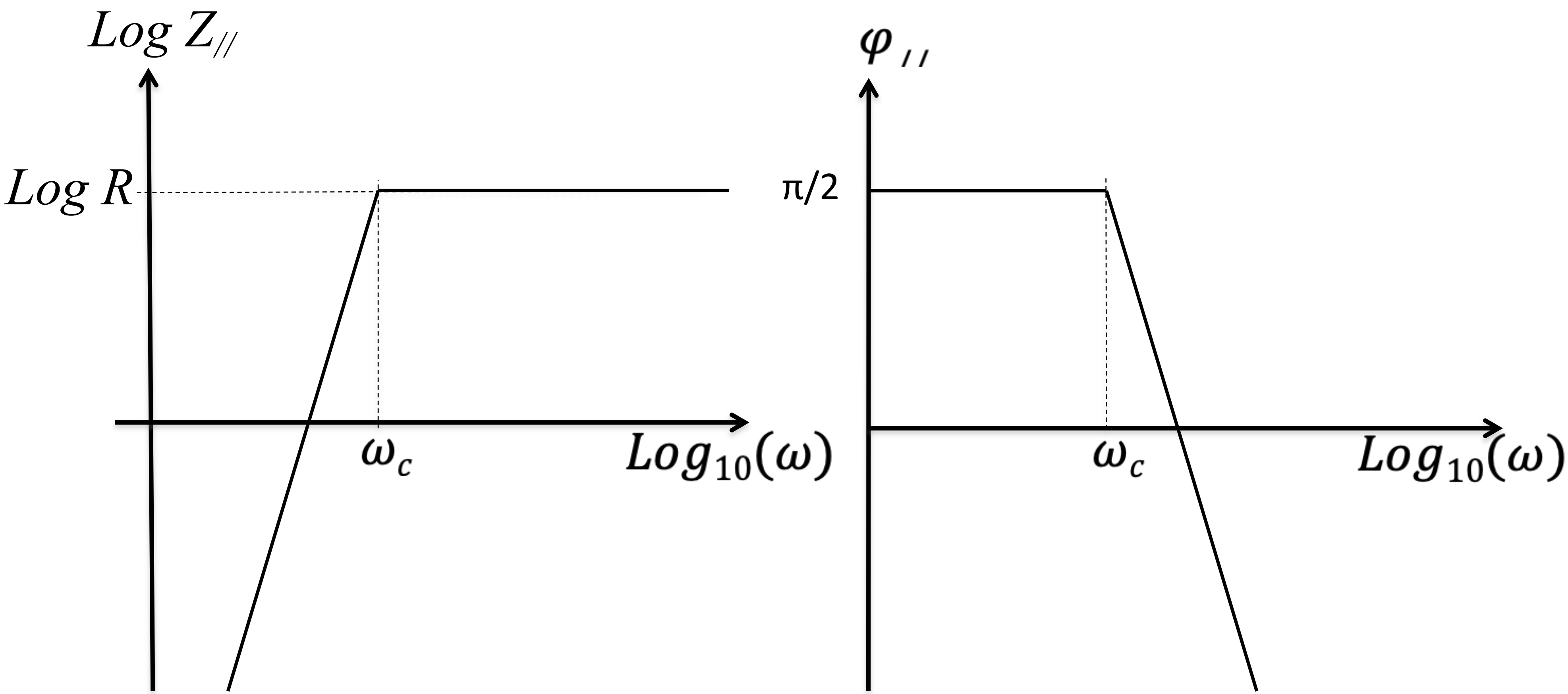
Equivalent Impedance

$$\underline{Z}_{//}(\omega) \equiv Z_{//}e^{j\varphi_{//}} = \frac{j\omega RL}{R + j\omega L} \begin{cases} Z_{//} = \dfrac{\omega RL}{\sqrt{R^2 + \omega^2 L^2}} \\ \varphi_{//} = \mathrm{arctg}\left(\dfrac{R}{\omega L}\right) \end{cases}$$

This impedance presents both

Amplitude and Phase

Bode Plots



The Bode plots show amplitude and phase of the impedance

Equivalent Impedance

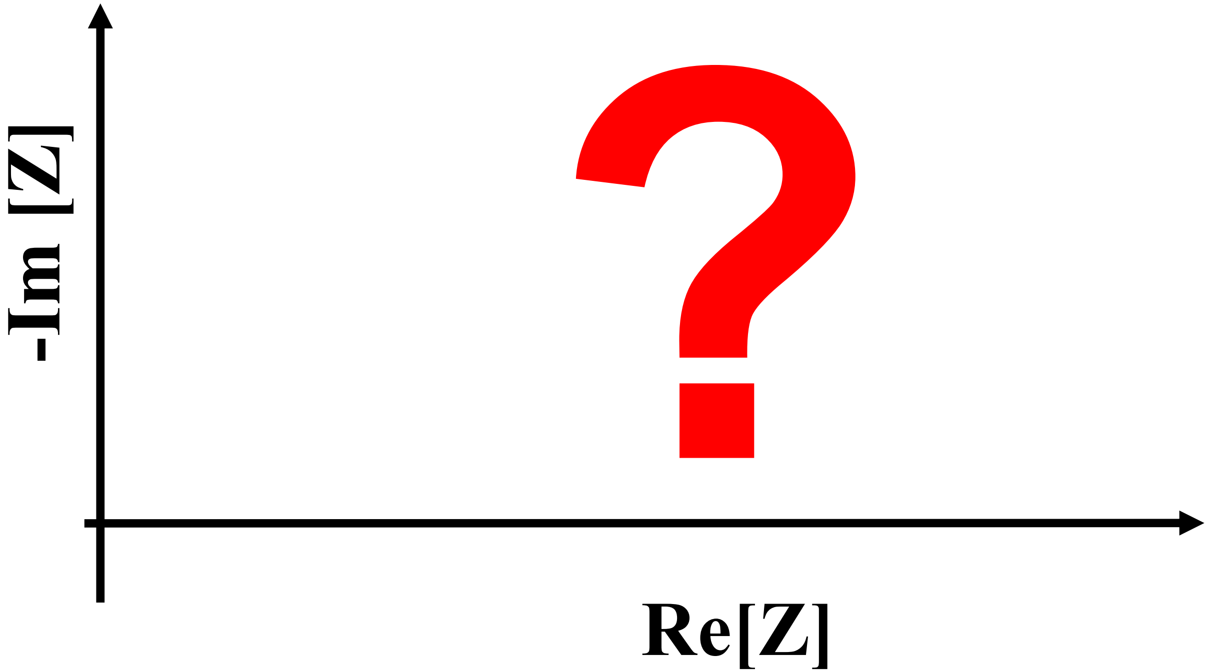
$$Z = \frac{R_L}{j\omega C_{DL} R_L + 1} \cdot \frac{1 - j\omega C_{DL} R_L}{1 - j\omega C_{DL} R_L}$$

$$Z = \frac{R_L - j\omega C_{DL} R_L^2}{1 + (\omega C_{DL} R_L)^2}$$

$$Z = \frac{R_L}{1 + (\omega C_{DL} R_L)^2} - j \frac{\omega C_{DL} R_L^2}{1 + (\omega C_{DL} R_L)^2} \quad \begin{cases} Z_{\mathrm{Re}} = \frac{R_L}{1+(\omega C_{DL} R_L)^2} \\ Z_{\mathrm{Im}} = -\frac{\omega C_{DL} R_L^2}{1+(\omega C_{DL} R_L)^2} \end{cases}$$

This impedance presents both resistive and reactive components

Nyquist Plot



The Nyquist plot is also a mean to fit data about a specific electrochemical cell

Nyquist Plot

$$\underline{Z}_{//} = R_{//} + jX_{//}, \text{with} \begin{cases} y = -X_{//} = \dfrac{\omega R_{\mathrm{L}} C_{\mathrm{DL}}}{1 + (\omega R_{\mathrm{L}} C_{\mathrm{DL}})^2} \\ x = R_{//} = \dfrac{R_{\mathrm{L}}}{1 + (\omega R_{\mathrm{L}} C_{\mathrm{DL}})^2} \end{cases}$$

$$\left|\underline{Z}_{//}\right|^2 = \frac{R_{\mathrm{L}}^2 \left[1 + (\omega R_{\mathrm{L}} C_{\mathrm{DL}})^2\right]}{\left[1 + (\omega RLC_{DL})^2\right]^2} = \frac{R_{\mathrm{L}}^2}{\left[1 + (\omega RLC_{\mathrm{DL}})^2\right]} = R_{\mathrm{L}} R_{//}$$

Demonstration that it is a circle
in the Complex plane

Nyquist Plot

$$Z_{//}^2 = R_{//}^2 + X_{//}^2 = R_L R_{//}$$

$$R_{//}^2 + X_{//}^2 - R_L R_{//} = 0$$

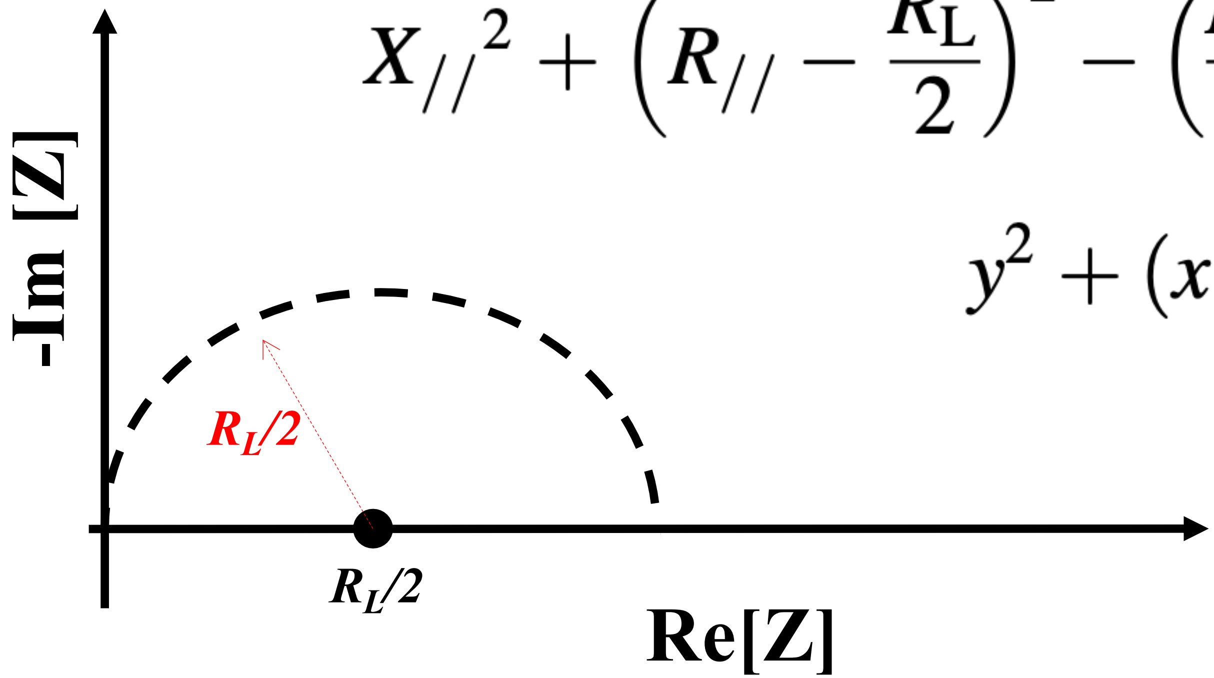
$$X_{//}{}^2 + \left(R_{//} - \frac{R_L}{2}\right)^2 - \left(\frac{R_L}{2}\right)^2 = 0$$

Demonstration that it is a circle
in the Complex plane

Nyquist Plot

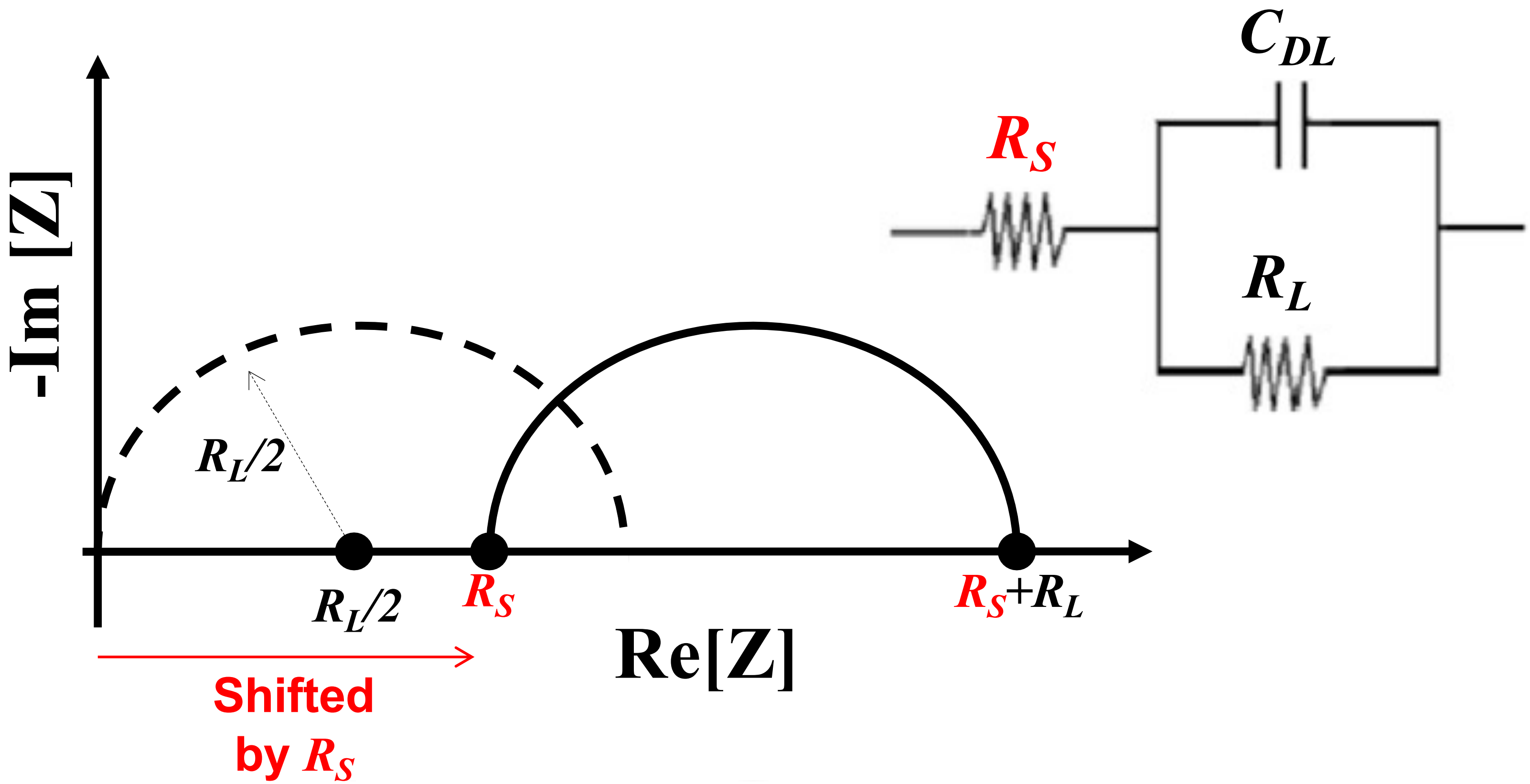
$$X_{//}^2 + \left(R_{//} - \frac{R_L}{2}\right)^2 - \left(\frac{R_L}{2}\right)^2 = 0$$

$$y^2 + (x - x_0)^2 - r^2 = 0$$



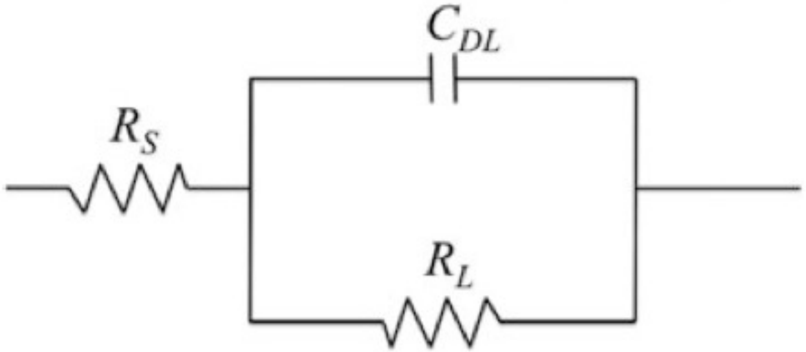
Demonstration that it is a circle
in the Complex plane

Nyquist Plot



That bring us actually to a circle shifted by R_s in the Complex plane

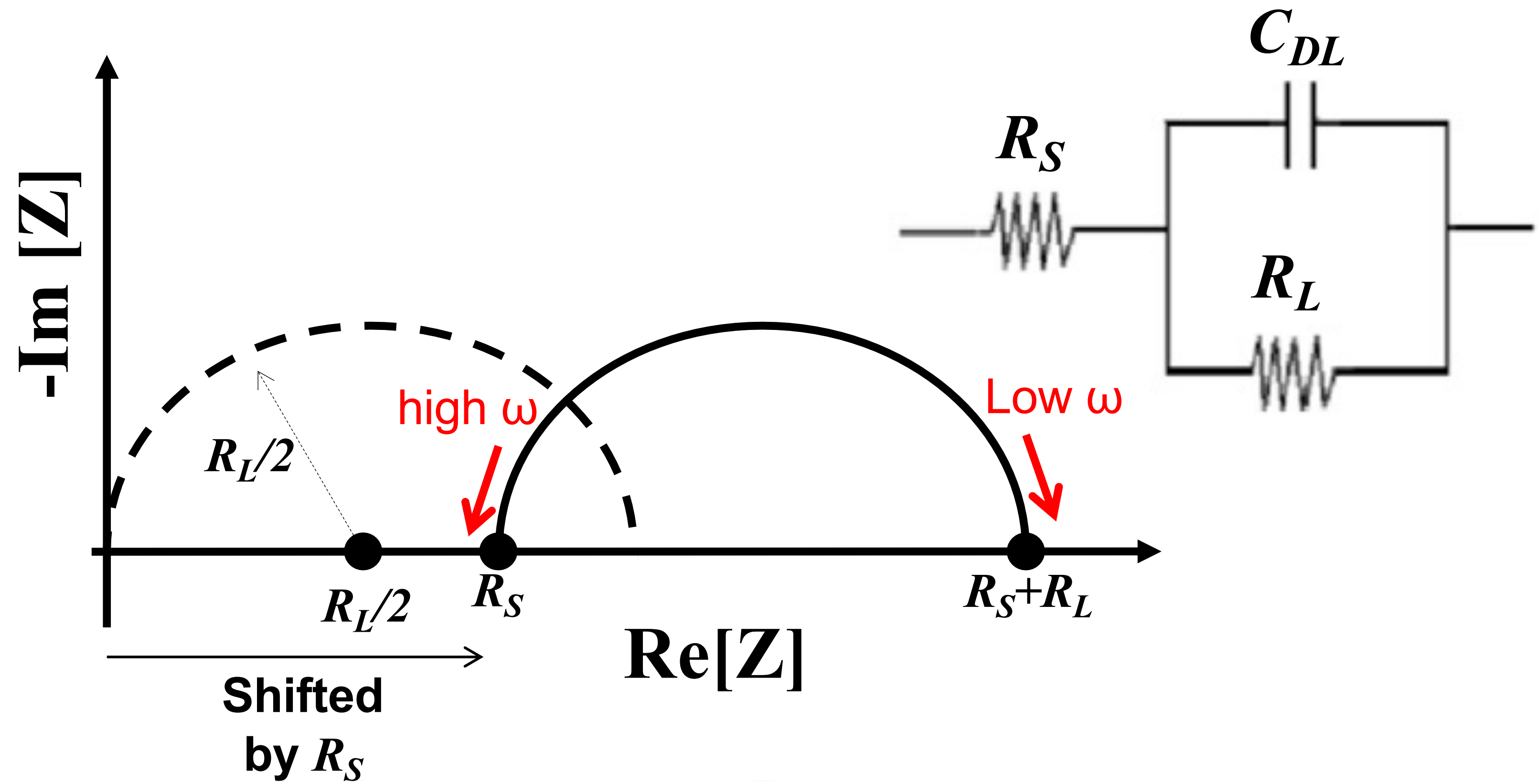
Equivalent Impedance



$$Z_{//} = Z = C_{DL} // R_L \quad \begin{cases} Z = \dfrac{R_L}{j\omega C_{DL} R_L + 1} \xrightarrow{\omega \to 0} R_L \\ Z = \dfrac{R_L}{j\omega C_{DL} R_L + 1} \xrightarrow{\omega \to \infty} 0 \end{cases}$$

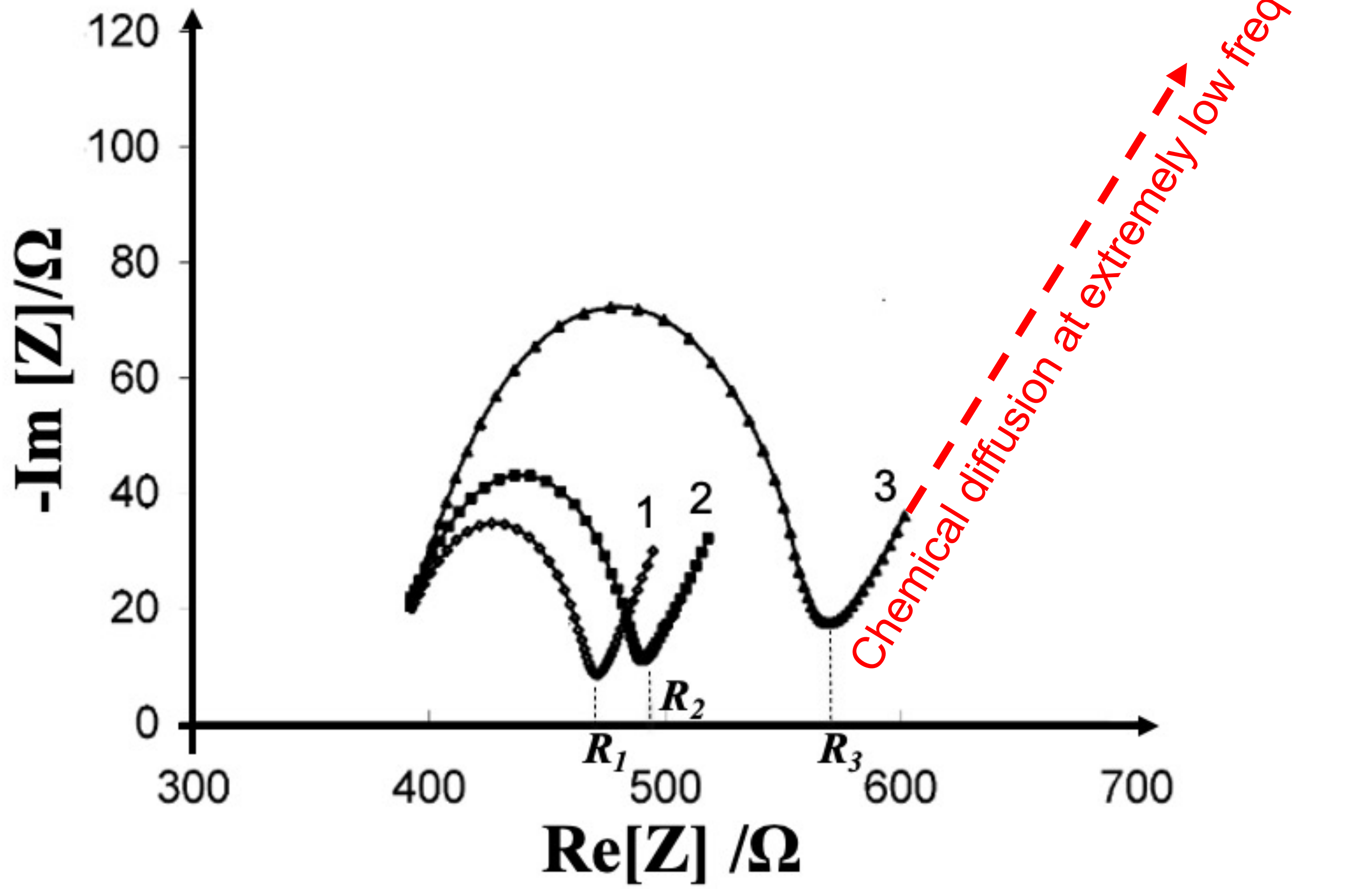
Frequency drive the move on the circle

Nyquist Plot



That bring us actually to a circle shifted by R_s in the Complex plane

Nyquist Plots
(also named Cole-Cole Plots)



Capacitance vs Frequency

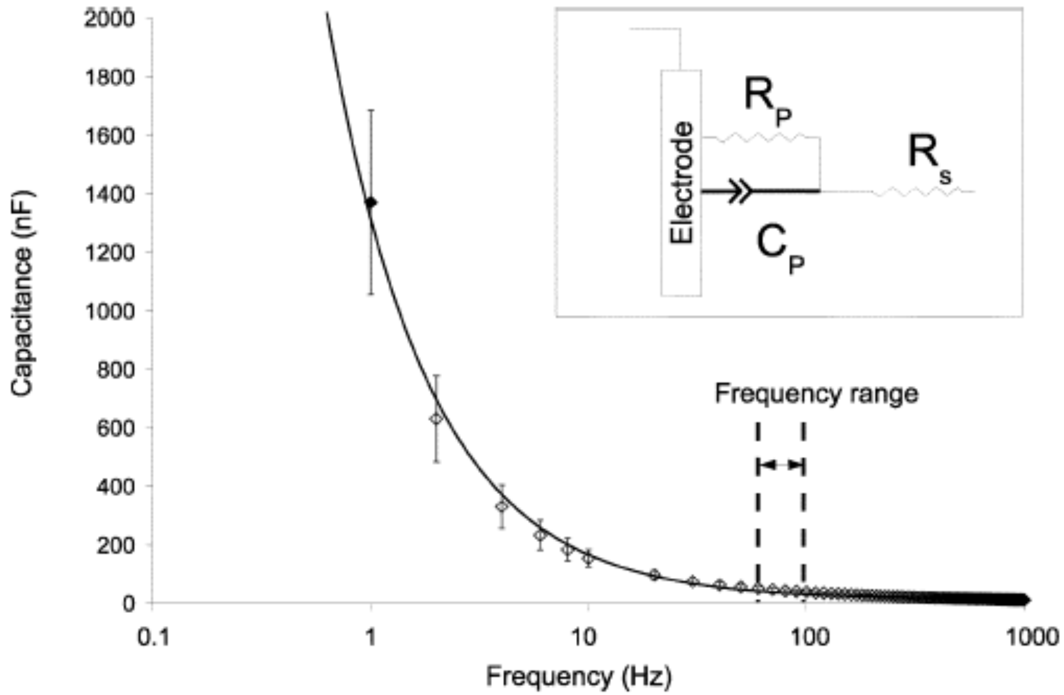
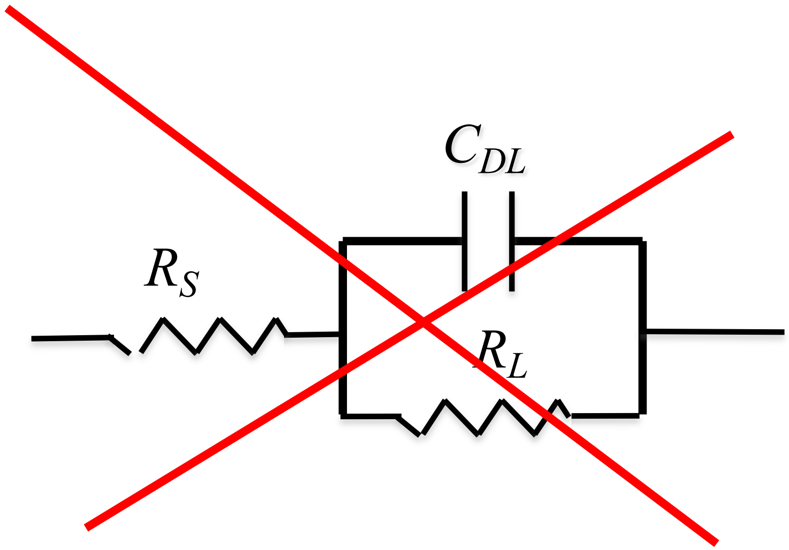
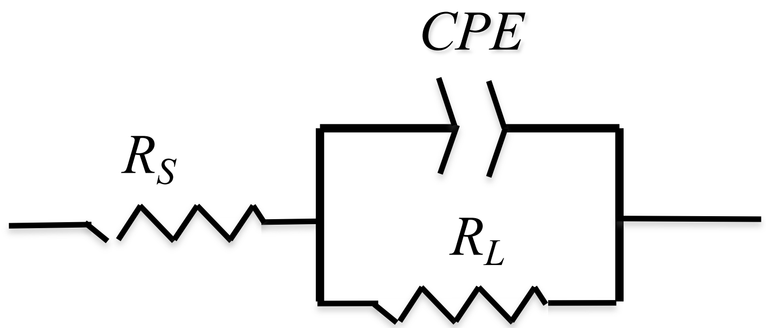


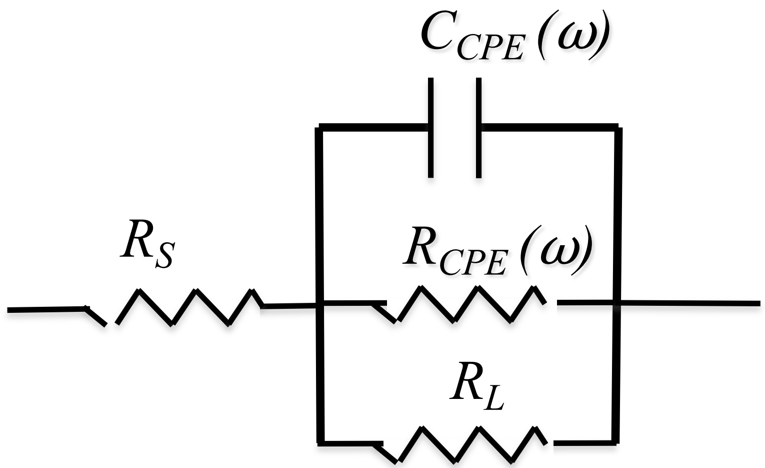
Fig. 9. Measured capacitance versus charge/discharge frequency on clean gold electrodes. The continuous line shows the fitting.

However, the Layering effect might correspond to a non-ideal capacitance

Interface models







Equivalent circuits for non-ideal layering effects

CPE element

$$Z_{CPE} = \frac{1}{C_p(j\omega)^\alpha} = \frac{\cos\left(\frac{\pi}{2}\alpha\right)}{C_p\omega^\alpha} - j\frac{\sin\left(\frac{\pi}{2}\alpha\right)}{C_p\omega^\alpha}$$

Only if $\alpha < 1$

$$Z_{CPE} \cong \frac{1}{\omega^\alpha C_p}\sqrt{1-\alpha^2} + \frac{1}{j\omega^\alpha C_p}\alpha.$$

The Constant Phase Element (CPE)

as Equivalent Component

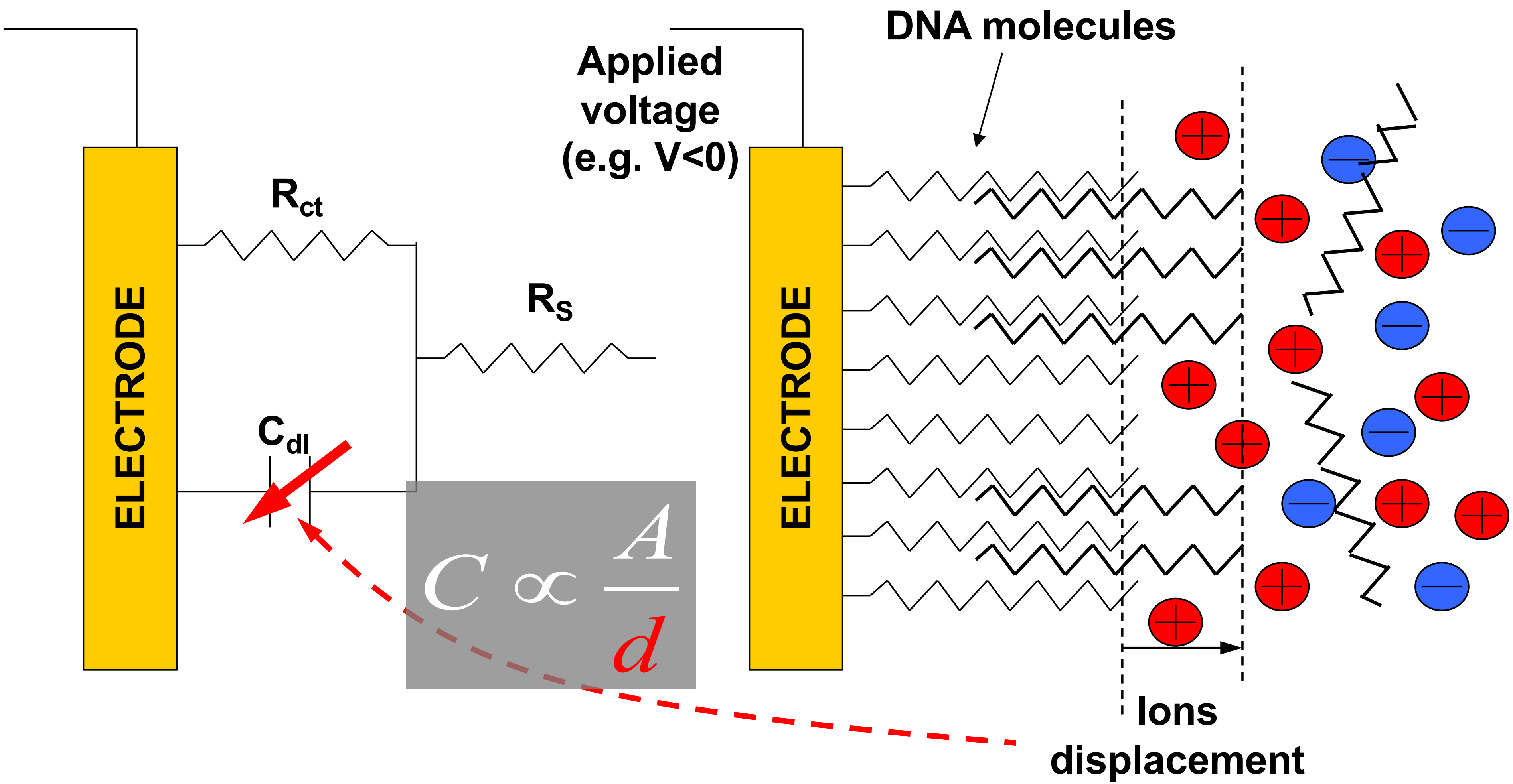
CPE element

$$Z_{CPE}\begin{cases} R_{CPE} \cong \frac{1}{\omega^{\alpha}C_p}\sqrt{1-\alpha^2} \\ X_{CPE} \cong \frac{-1}{\omega^{\alpha}C_p}\alpha \end{cases}$$

$$|X_{CPE}| \cong \frac{1}{\omega^{\alpha}C_p}\alpha = \frac{1}{\omega^{\alpha-1}\omega\, C_p}\alpha = \frac{1}{\omega\left(\frac{C_p}{\alpha\,\omega^{1-\alpha}}\right)}$$

$$C_{CPE} \cong \frac{C_p}{\alpha\,\omega^{1-\alpha}}$$

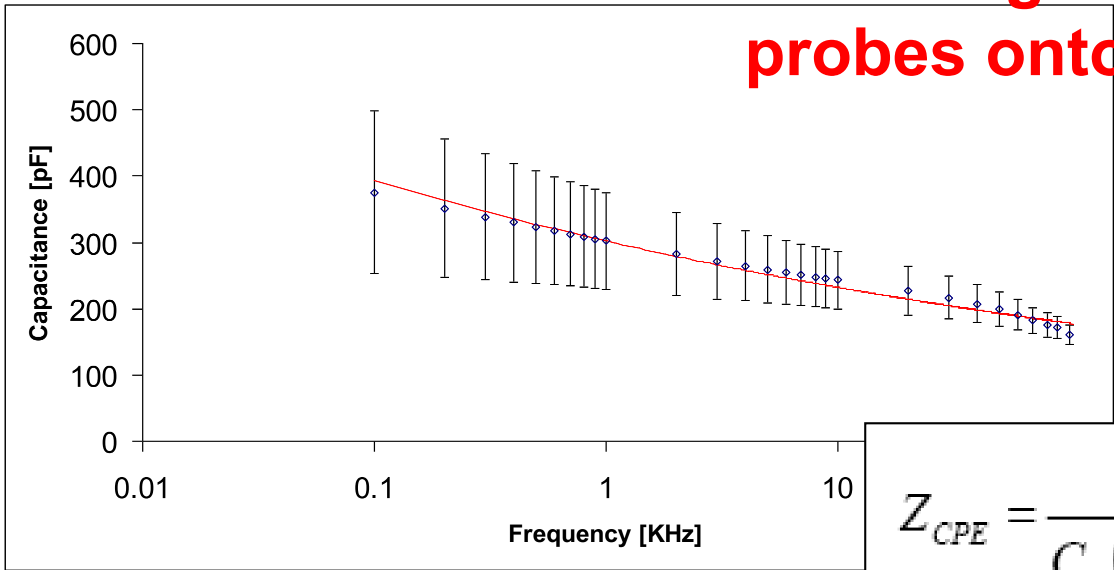
The Capacitance DNA Detection

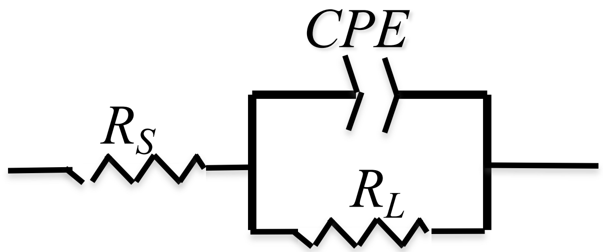


Unlabeled ssDNA may be detected with capacitance measurements as due to charge displacement

Frequency behavior in DNA sensing

DNA Targets hybridized to DNA probes onto Gold electrodes





@ 1KHz

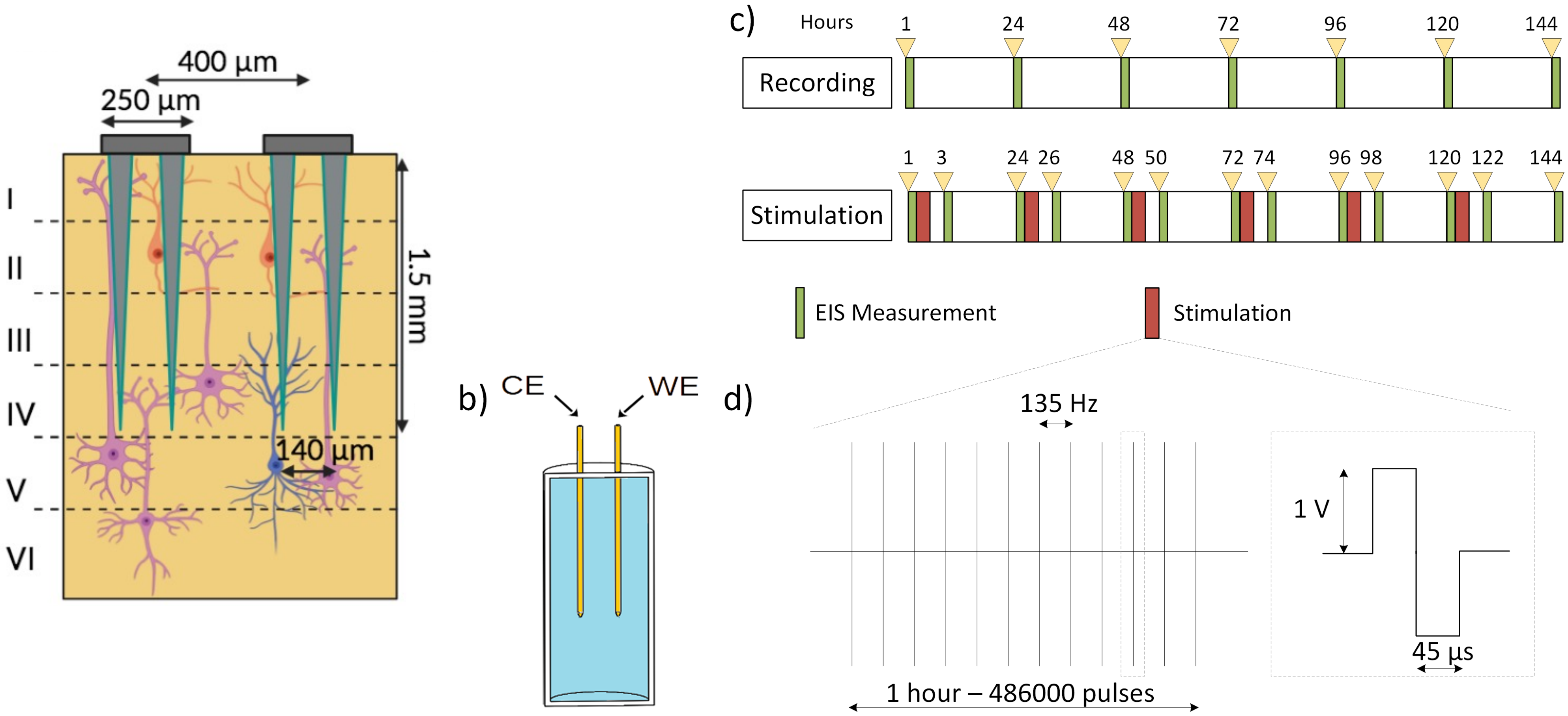
$$Z_{CPE} = \frac{1}{C_p (j\omega)^\alpha} \cong \frac{1}{\omega^\alpha C_p}\sqrt{1-\alpha^2} - j\frac{1}{\omega^\alpha C_p}\alpha$$

CPE parameters	Bare electrode	DNA Probes	DNA Target
R(GΩ)	45	177	177
X(GΩ)	123	837	970

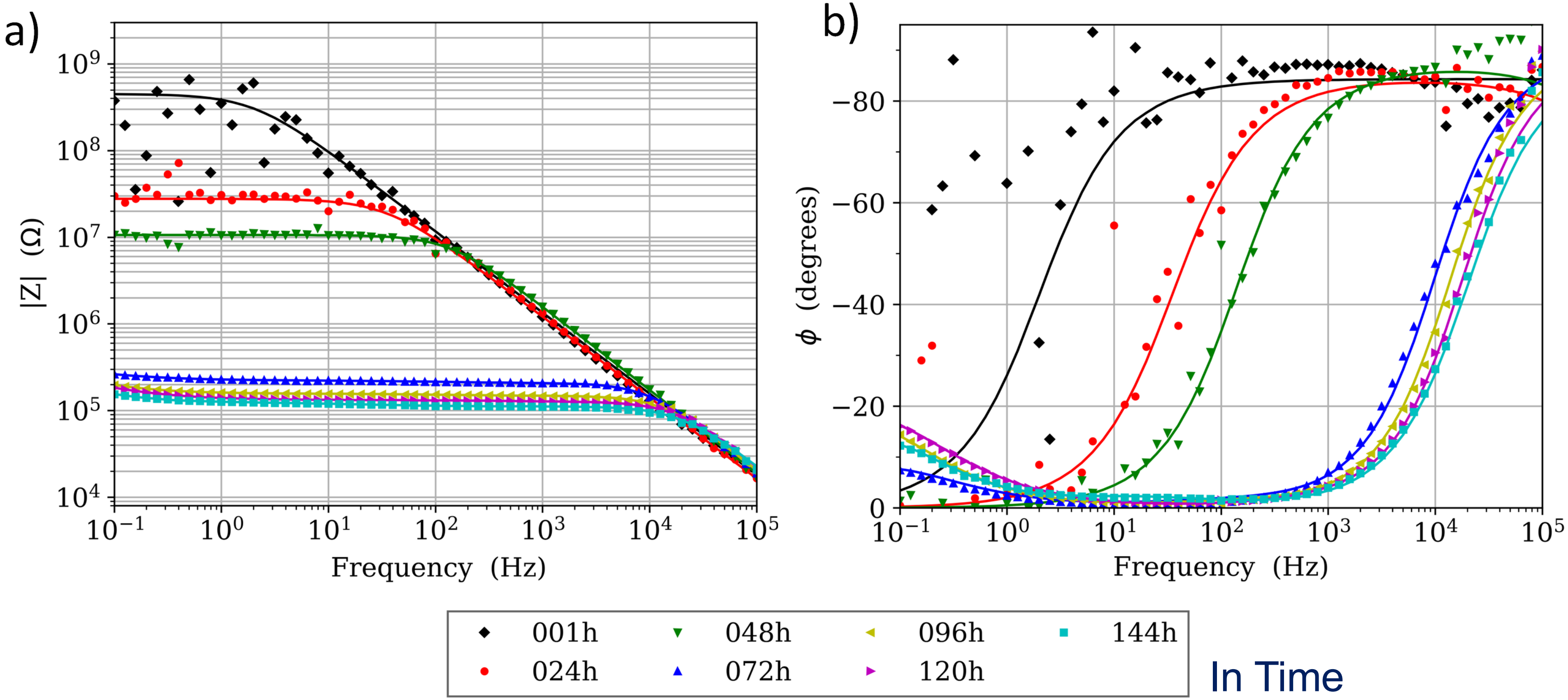
S.Carrara et al., Sensors and Transducer Journal 76 (2007) 969-977

From ssDNA to dsDNA, the R_L does not change, nor the R_{CPE}, while the X_{CPE} does

Brain/CMOS interfaces



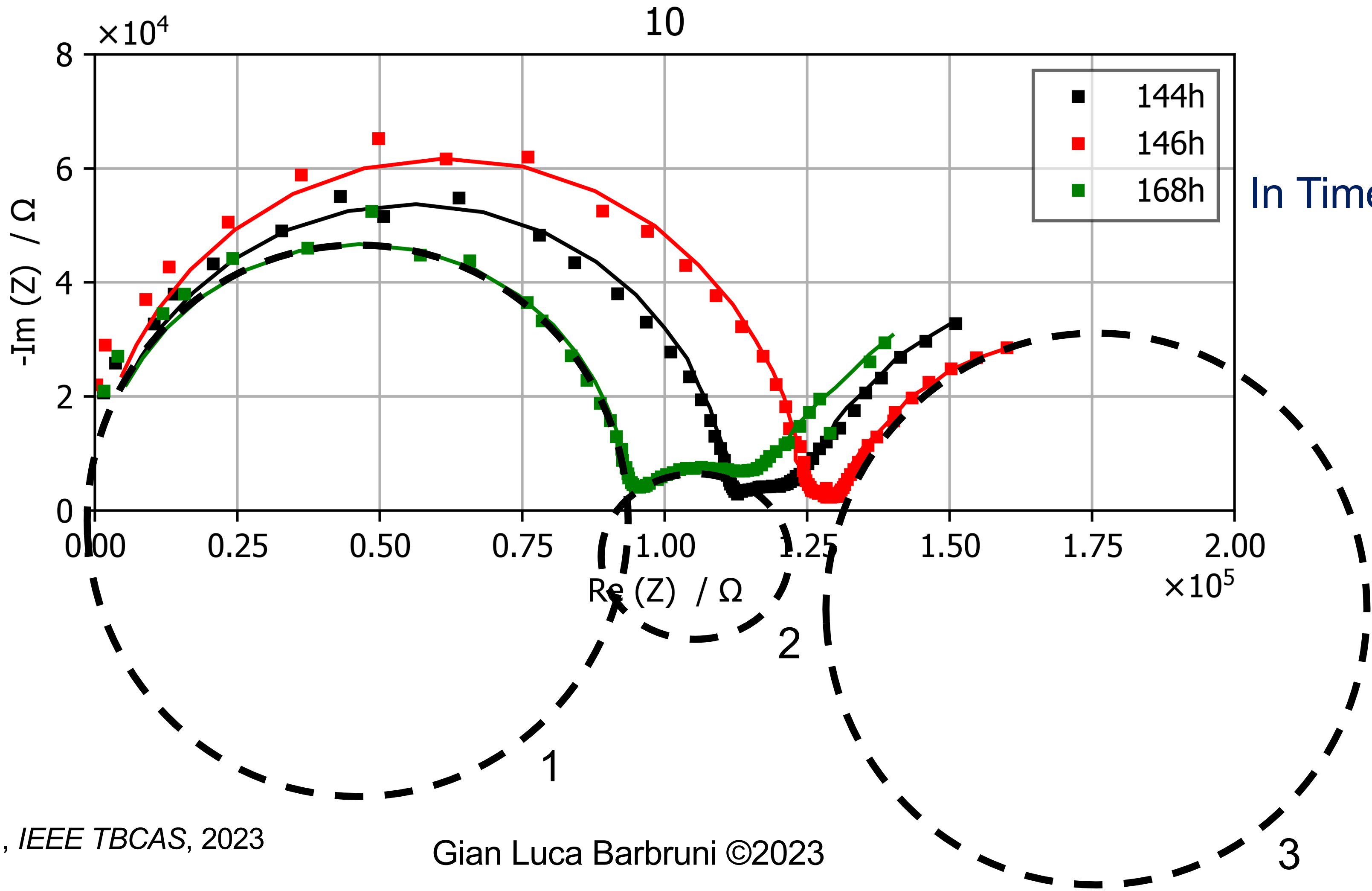
Bode Plots on Brain/CMOS interfaces



In Time

Barbruni et al., *IEEE TBCAS*, 2023

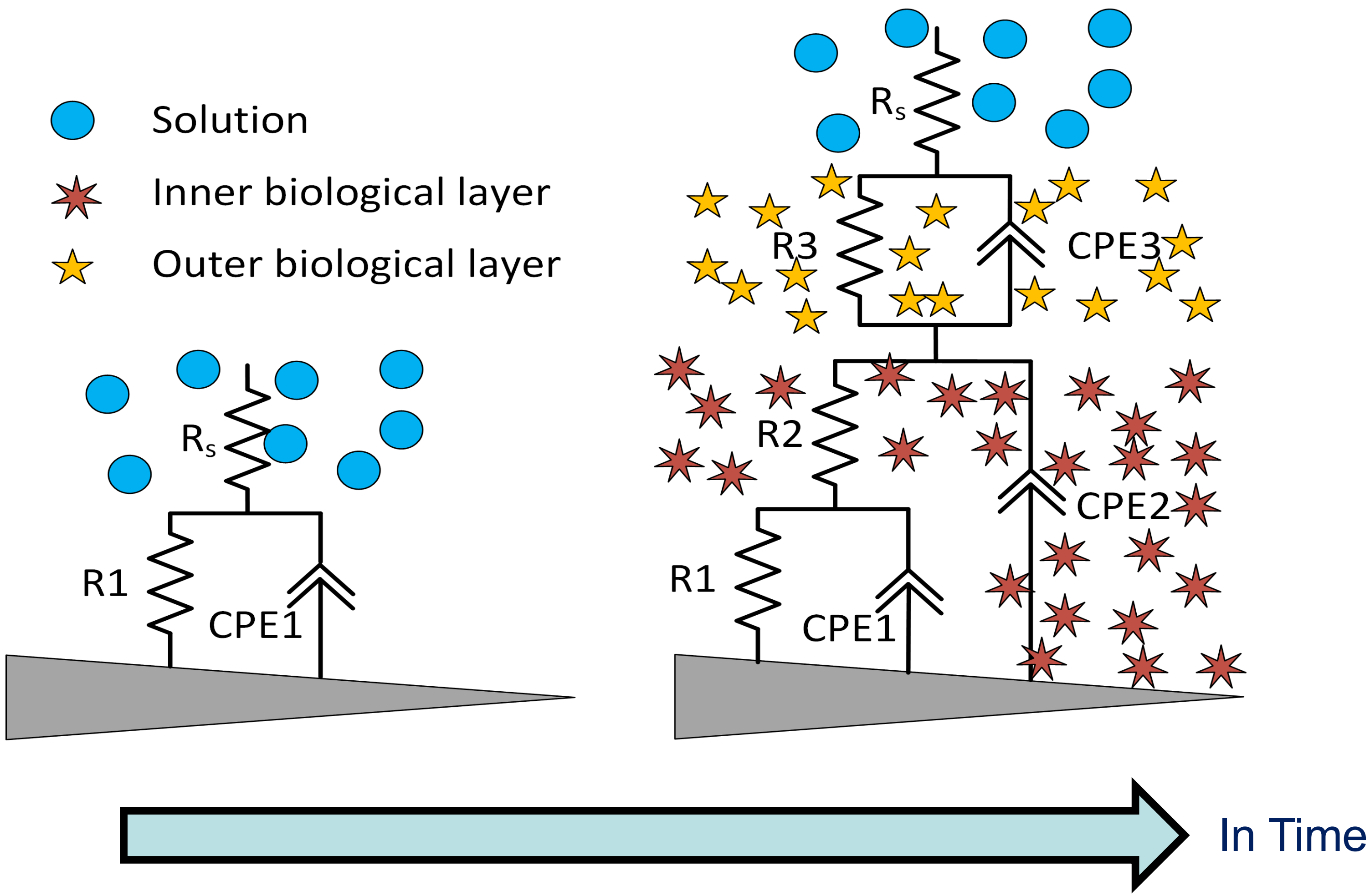
Nyquist Plots on Brain/CMOS interfaces



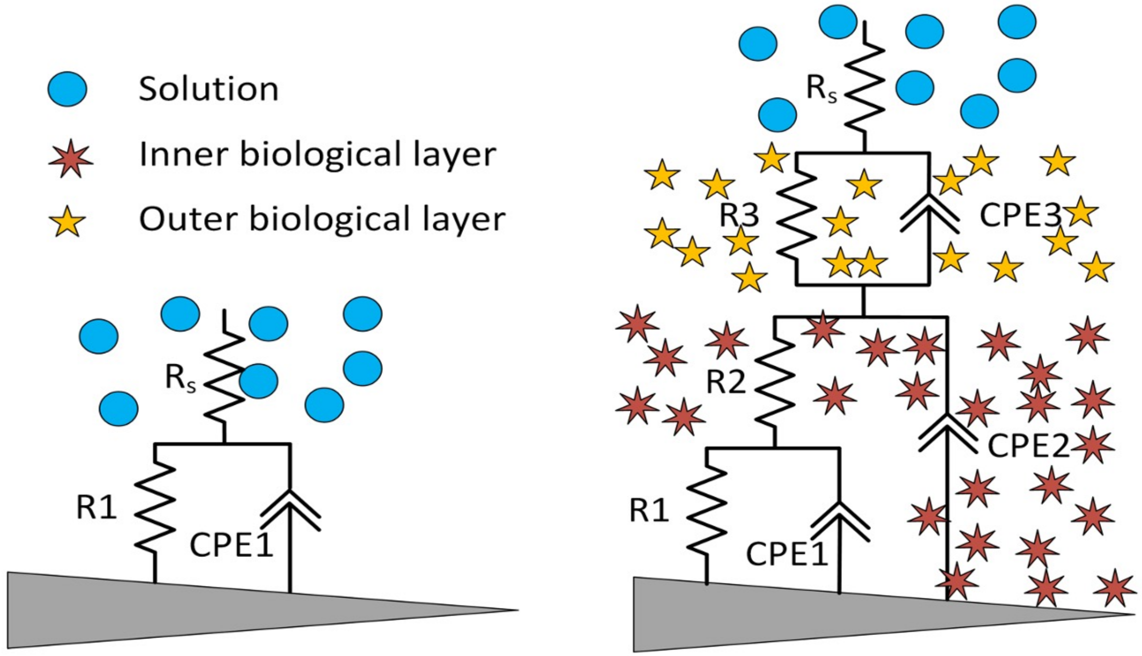
In Time

Barbruni et al., *IEEE TBCAS*, 2023

Better Model for Brain/CMOS interfaces

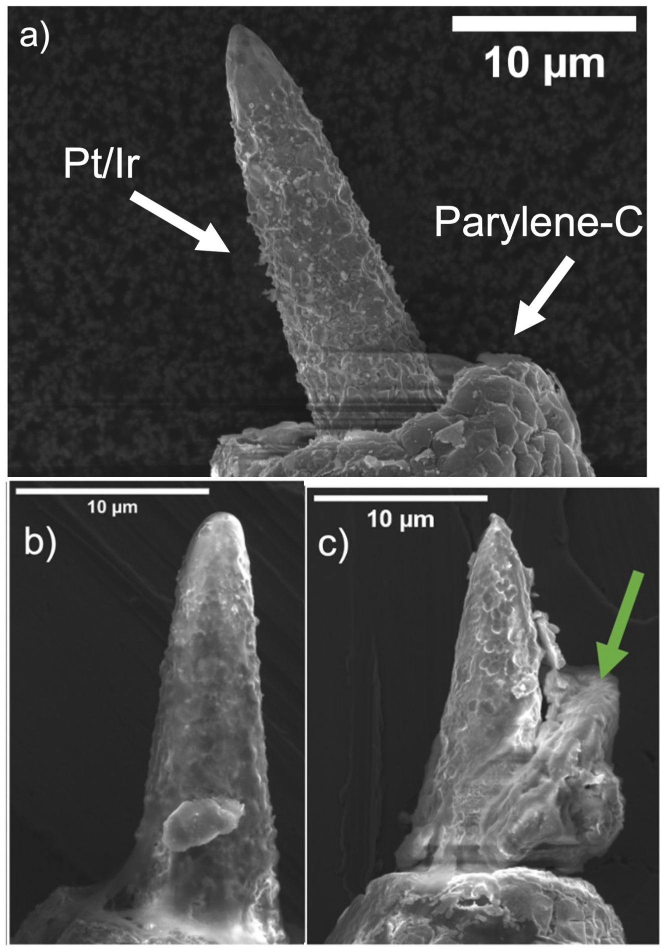


Brain/CMOS interfaces



Barbruni et al., *IEEE TBCAS*, 2023

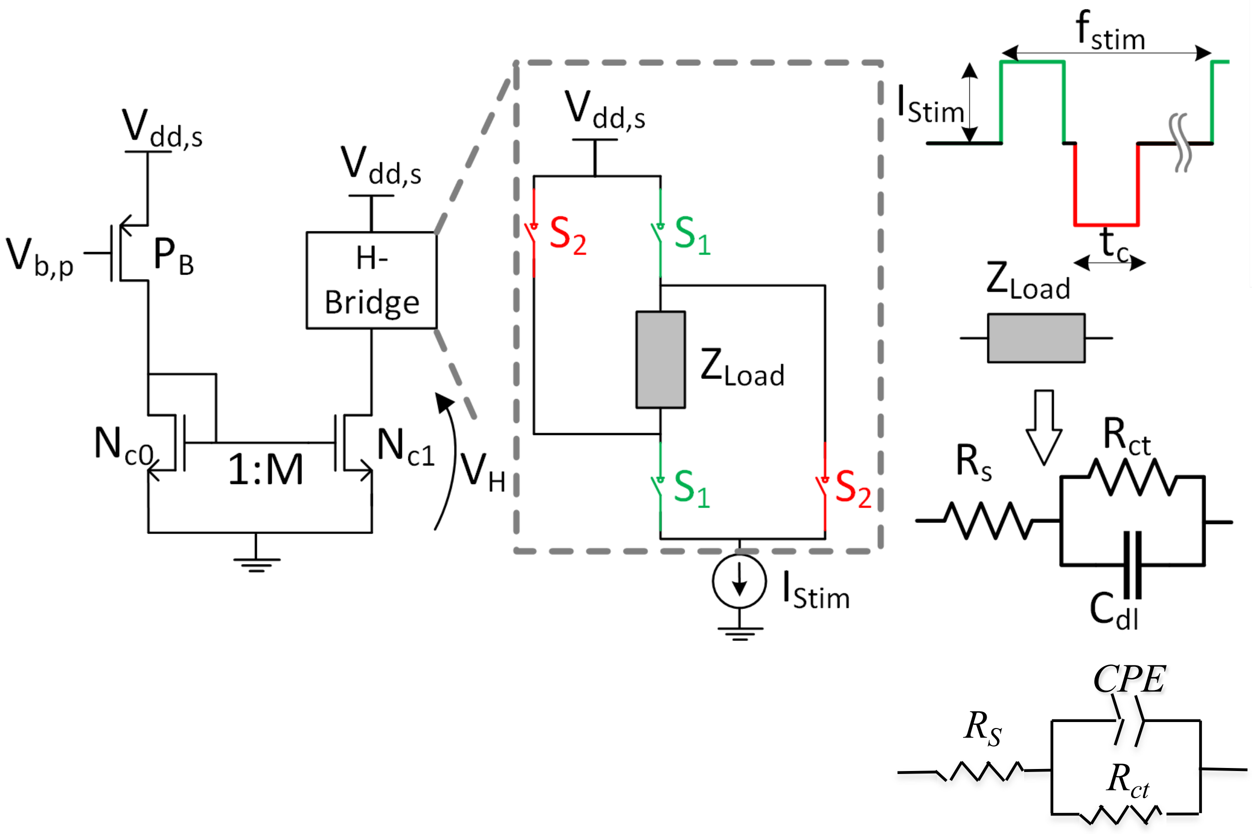
Initial

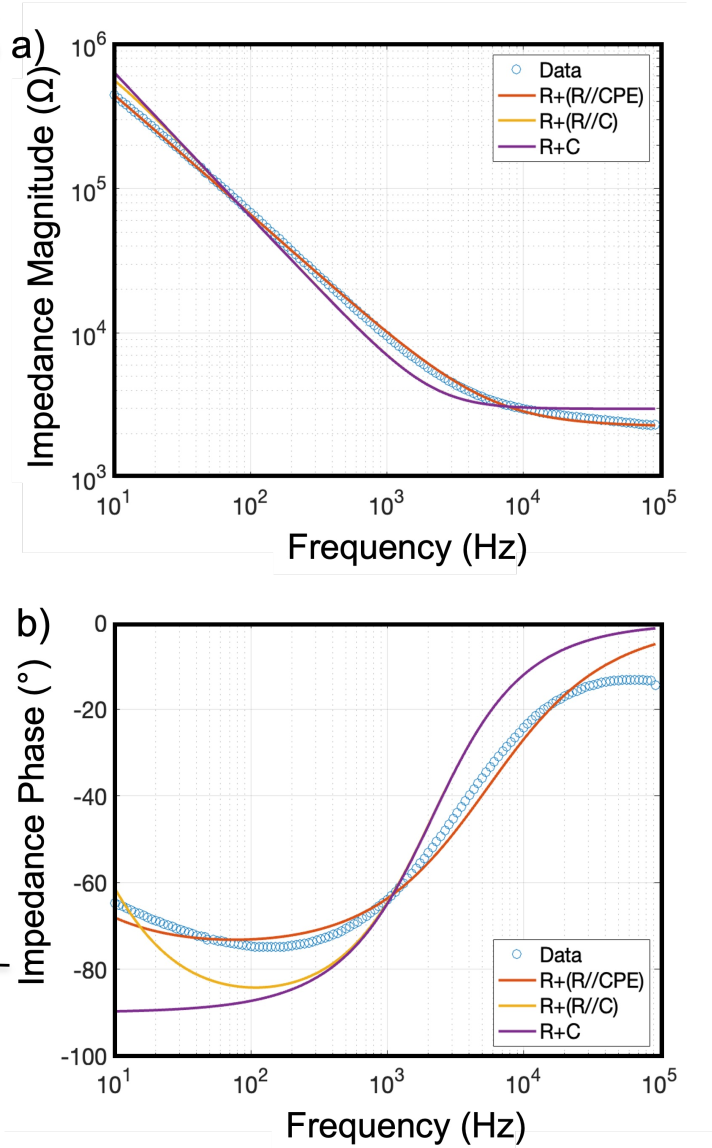


After Recording

Back to clean After Stimulation

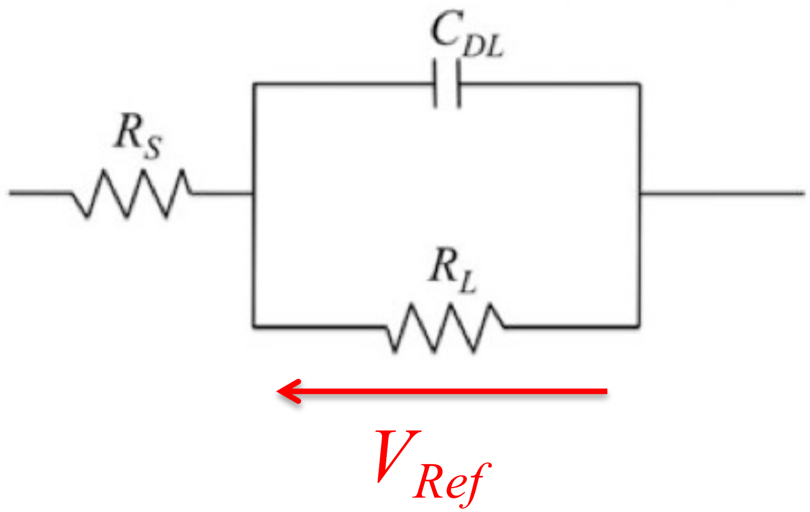
Different Models for Neurostimulators





Barbruni et al., *IEEE MOCAST*, 2023

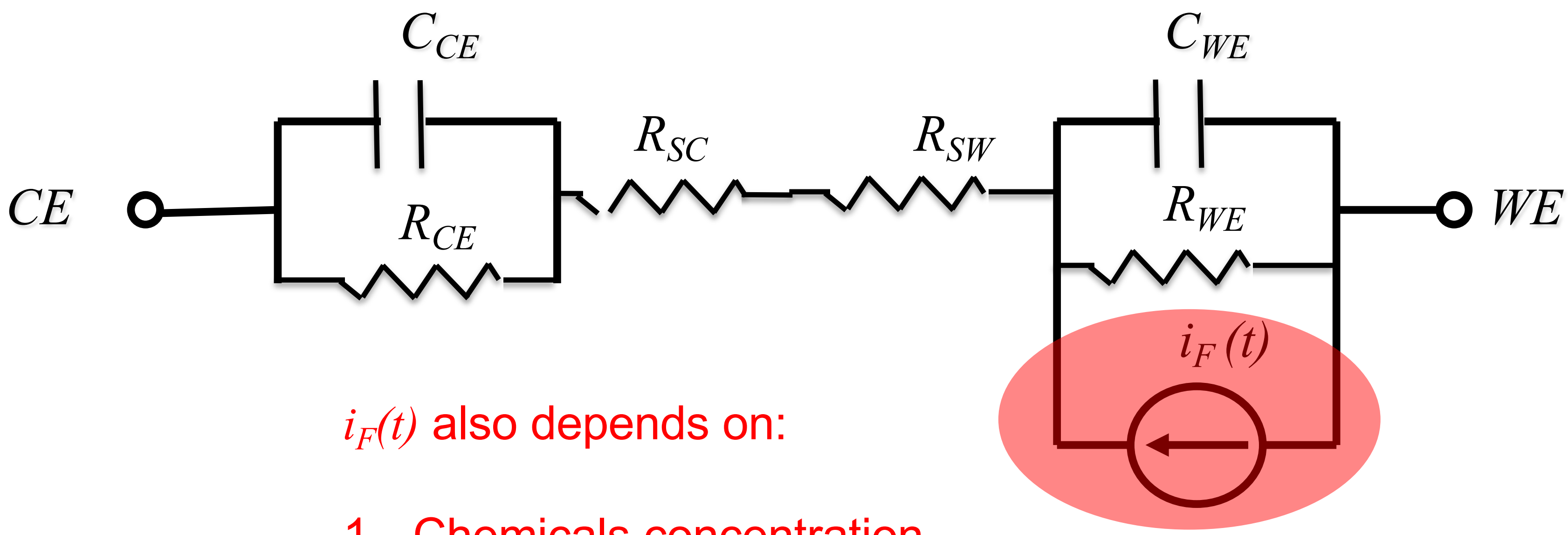
Non-Faradaic Current: passive model



$$I_{non-F} = \frac{V_{ref}}{Z} = \frac{1 + j\omega C_{DL} R_L}{R_L} V_{ref}$$

Non-Faradaic currents are also circulating in the cell

Faradaic Current: active model

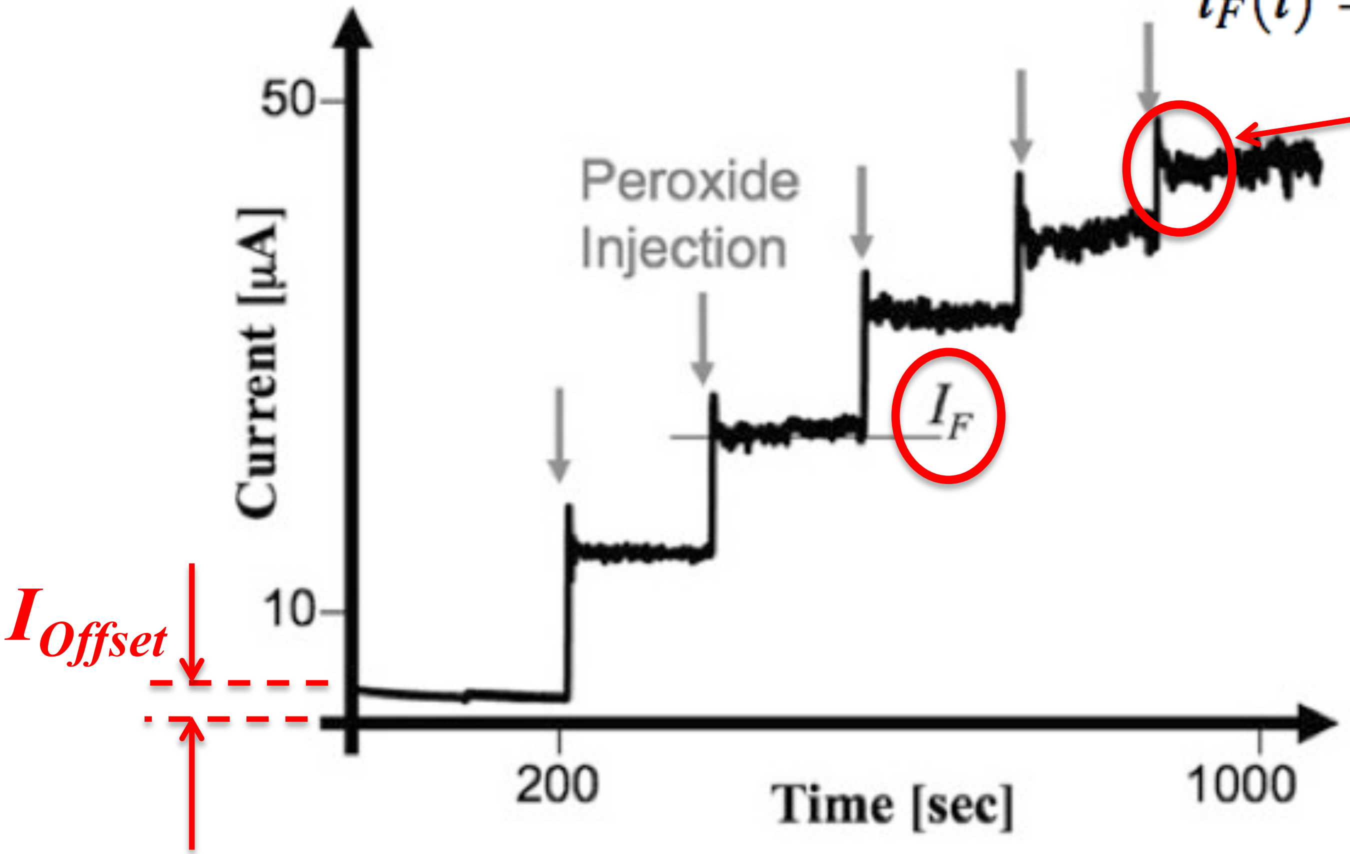


$i_F(t)$ also depends on:

1. Chemicals concentration
2. Time
3. Applied Voltage

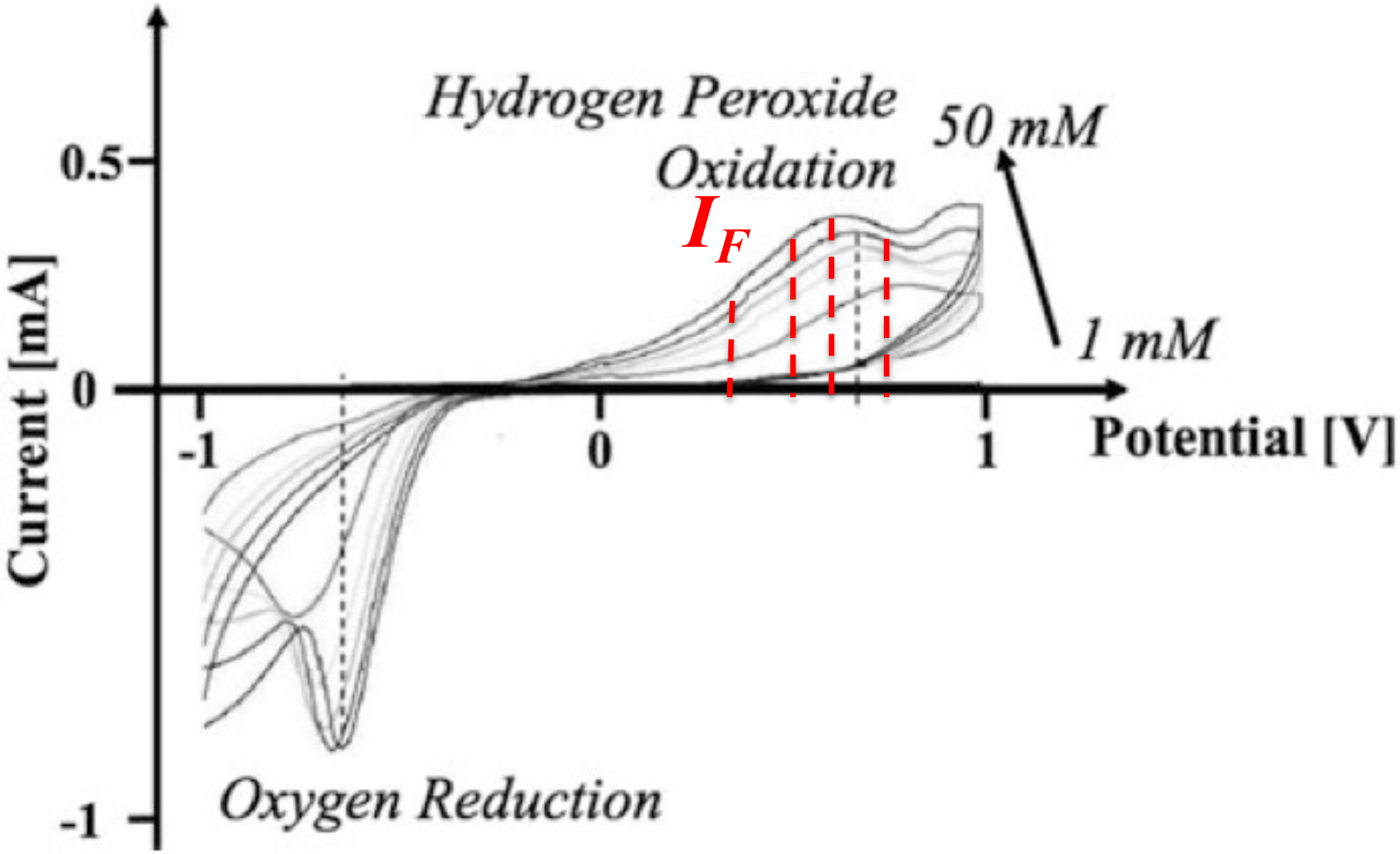
Faradaic Current versus Time & Concentration

$$i_F(t) = \frac{nFA\sqrt{D}C(t)}{\sqrt{\pi t}}$$



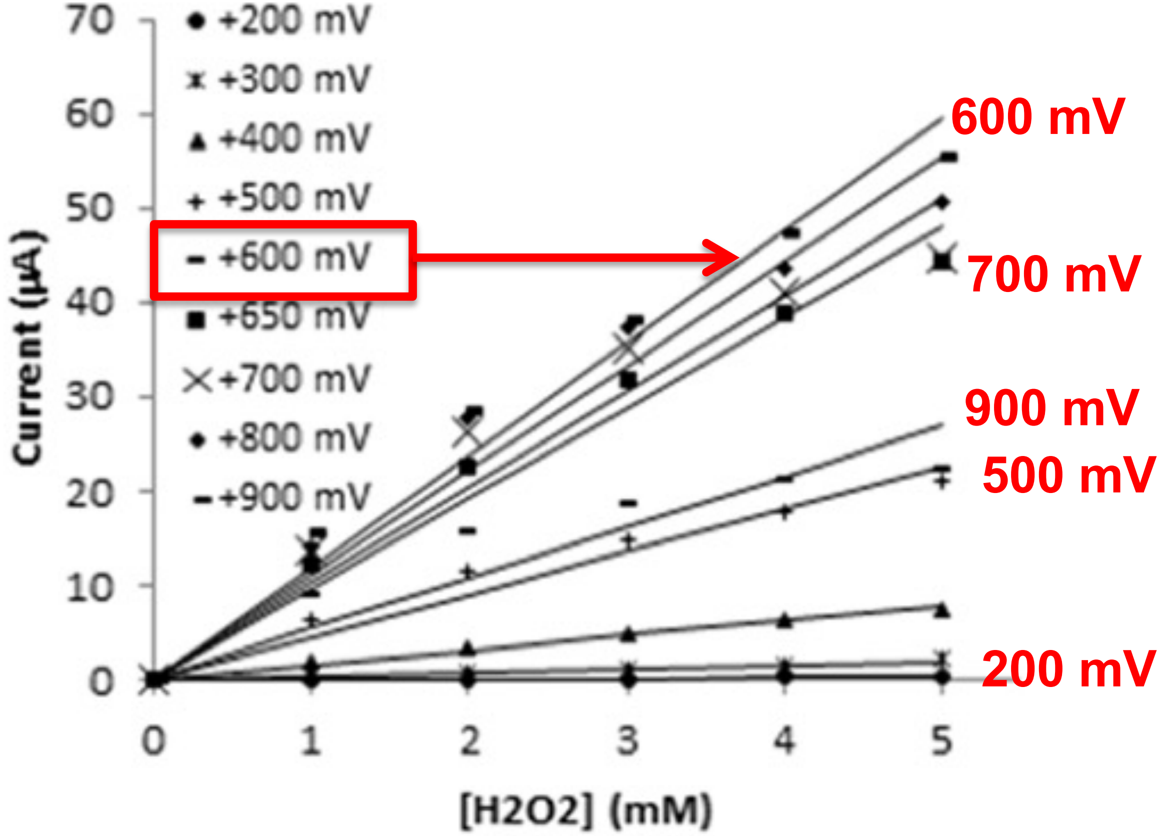
Typical curve in chronoamperometry

Faradaic Current versus Voltage & Concentration



O+ reduction and H_2O_2 oxidation observed by potential sweeping

Faradaic Current Generator



The sensitivity depends on the Reference Potential

Faradaic Current Generator

$$I = SC + I_{Offset}$$

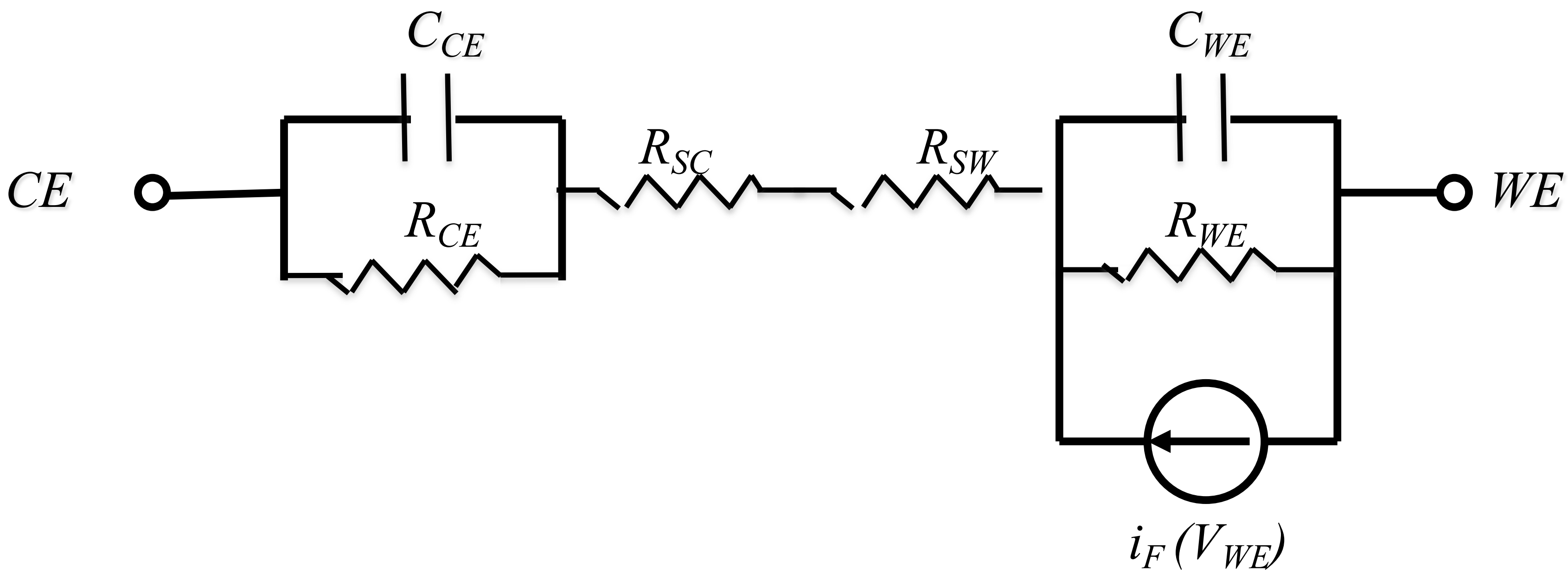
$$I_{Offset} = 0$$

$$I = SC$$

$$I(t,V) = S(V)C(t) \qquad S(V) = S_0 e^{-\frac{(V-V_0)^2}{\sigma}}$$

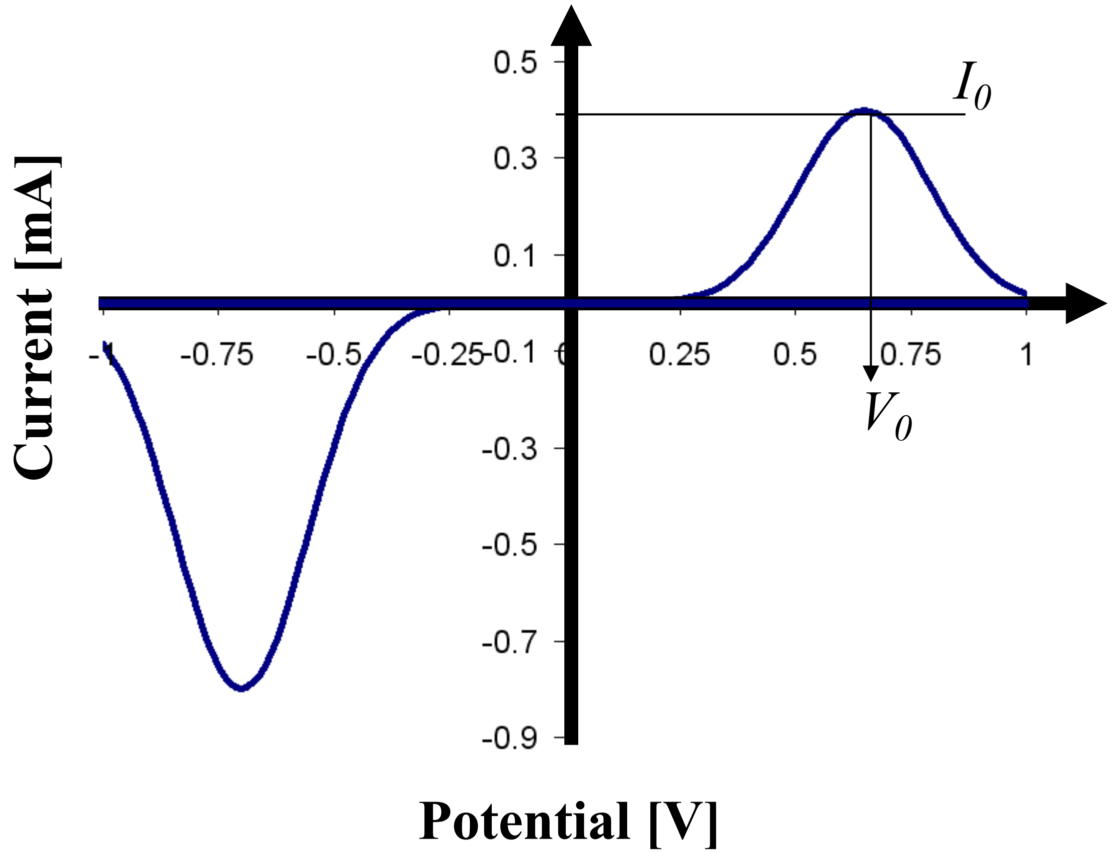
The sensitivity depends on the Reference Potential

Equivalent circuit



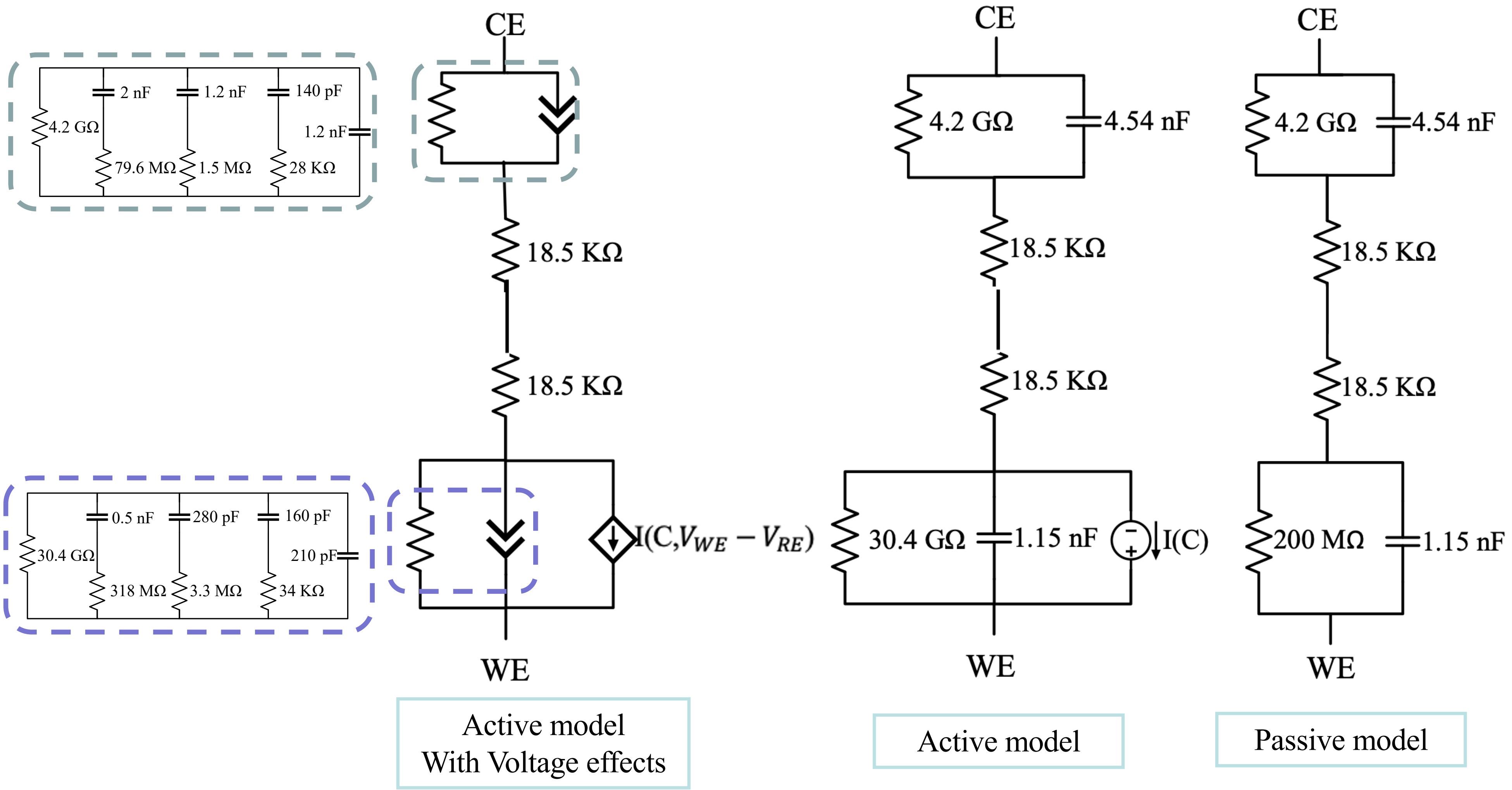
The equivalent circuit of the electrochemical cell needs to take into account that now the Faradaic current depends by voltage too

Faradaic currents on Voltage-scan

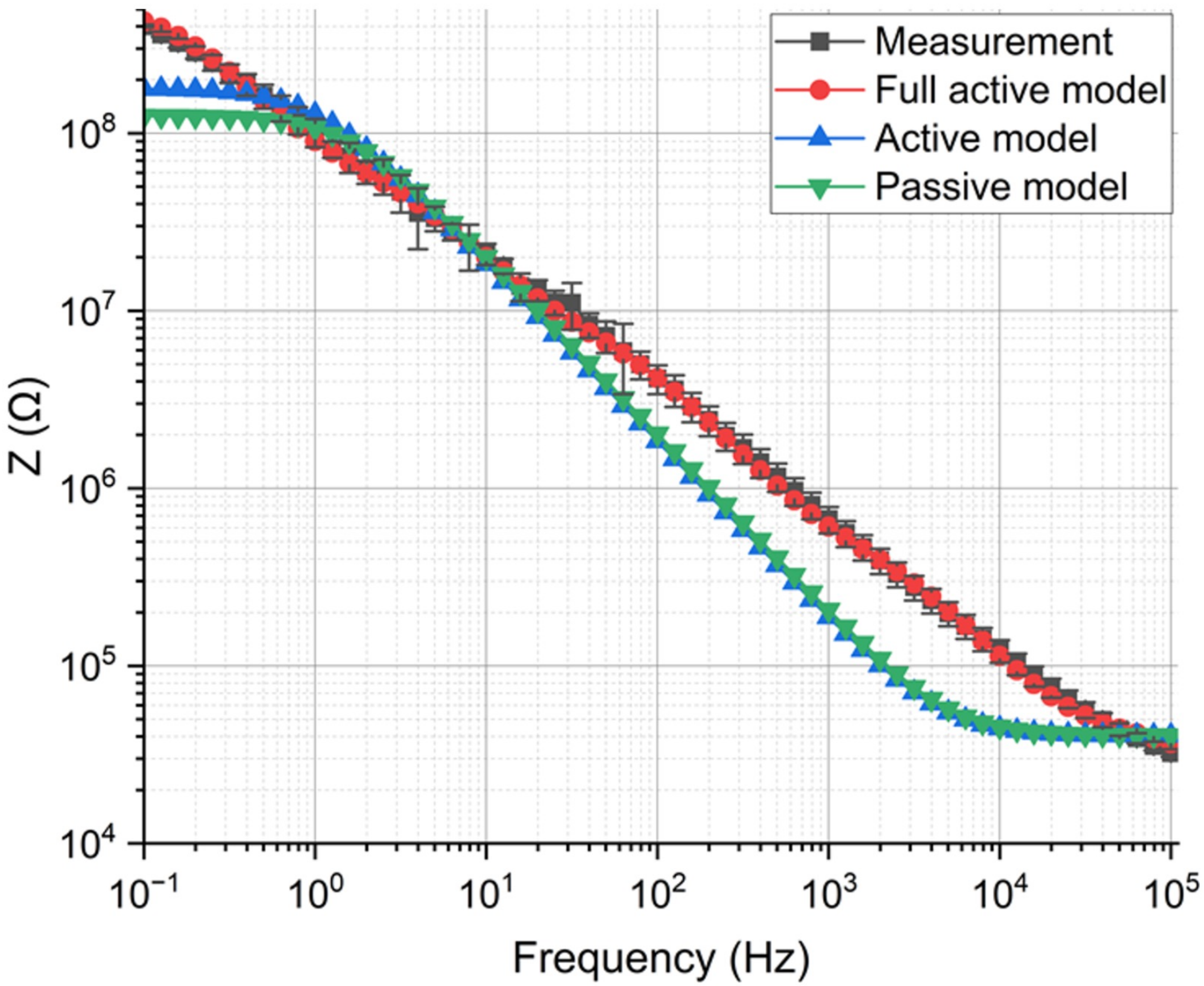


Faradaic currents with dependence by voltage may be simulated (in this case, without capacitance current)

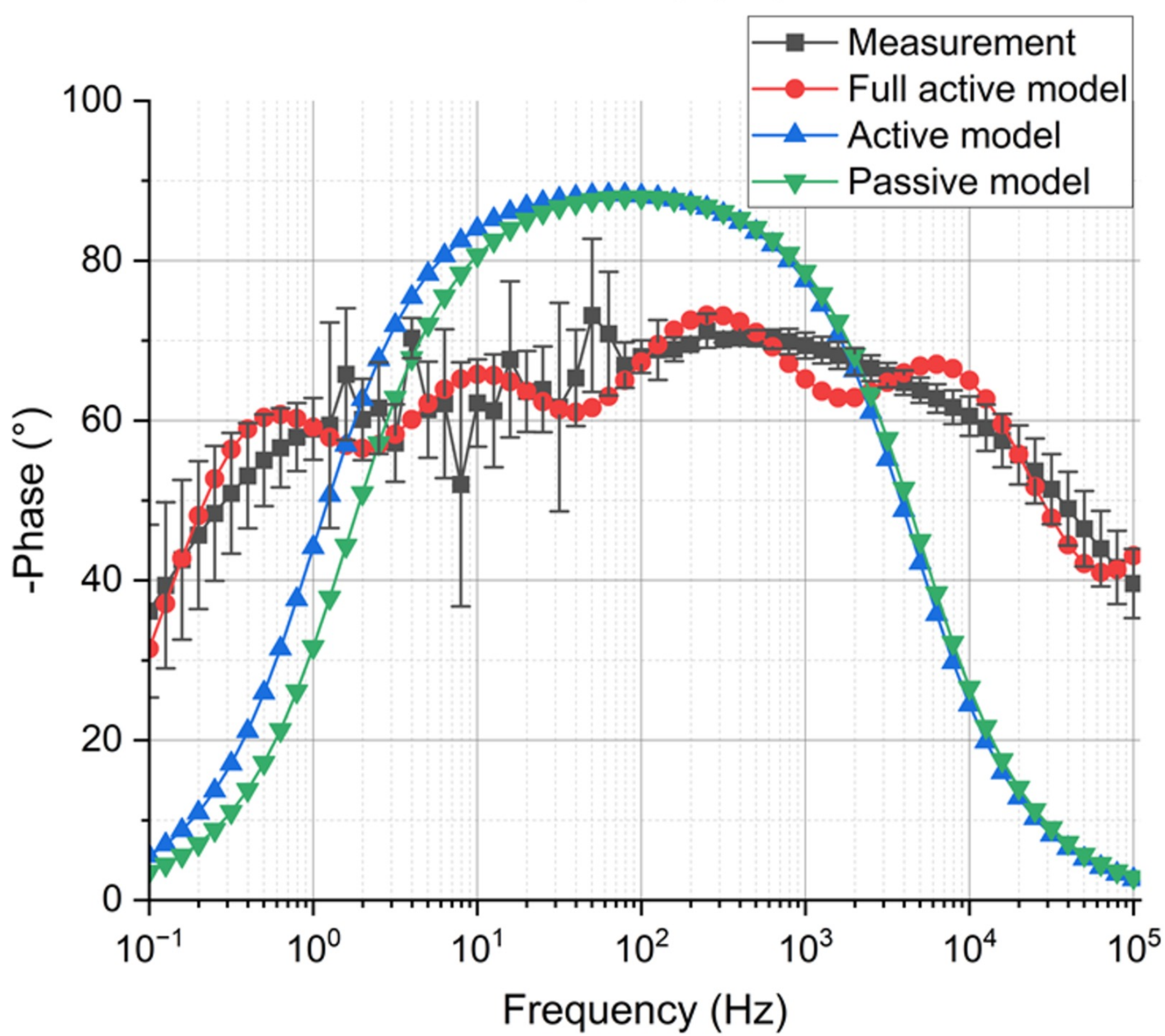
Equivalent Circuit for Bio/CMOS interfaces



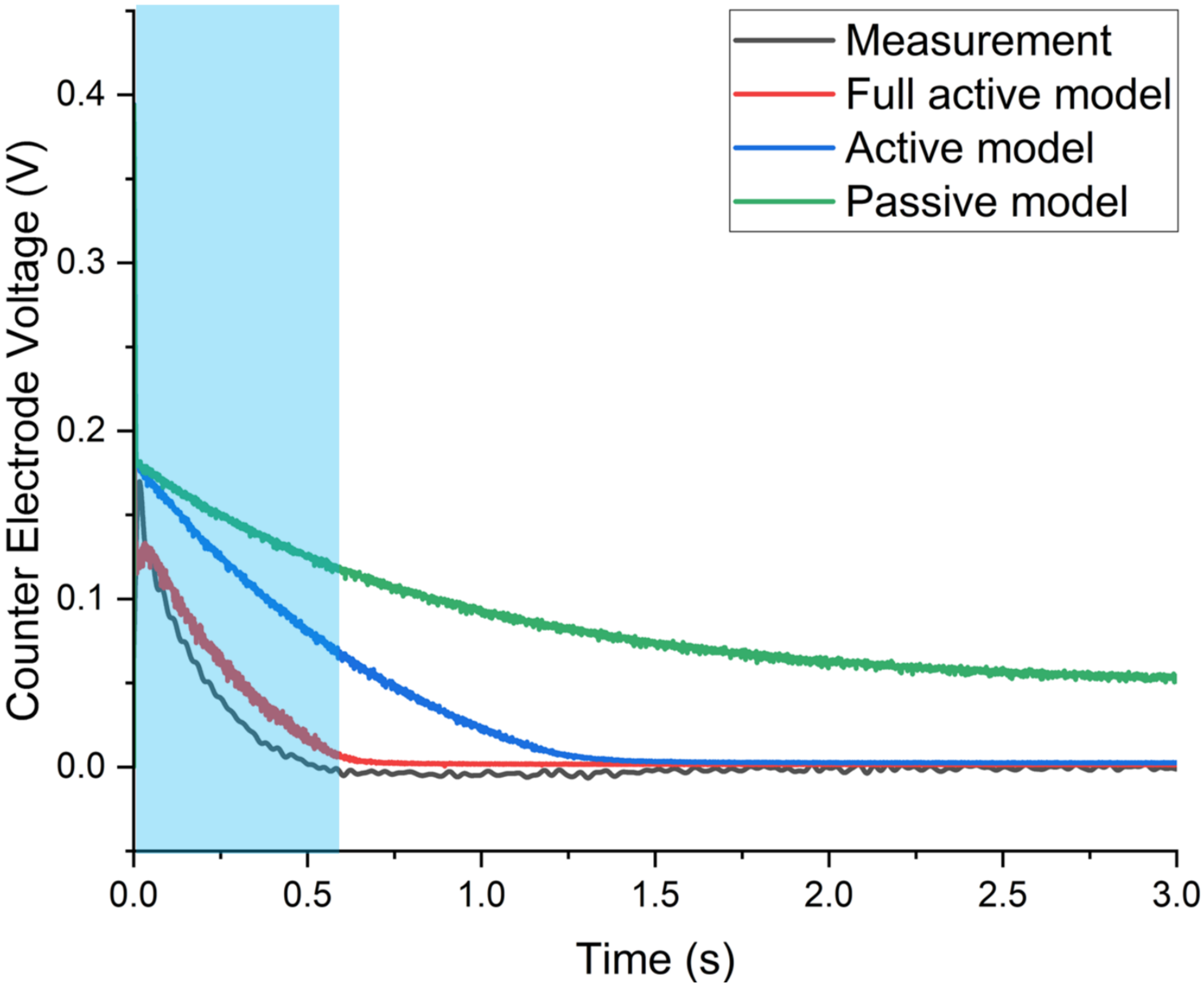
Impedance in Frequency of Bio/CMOS interfaces in micron sizes



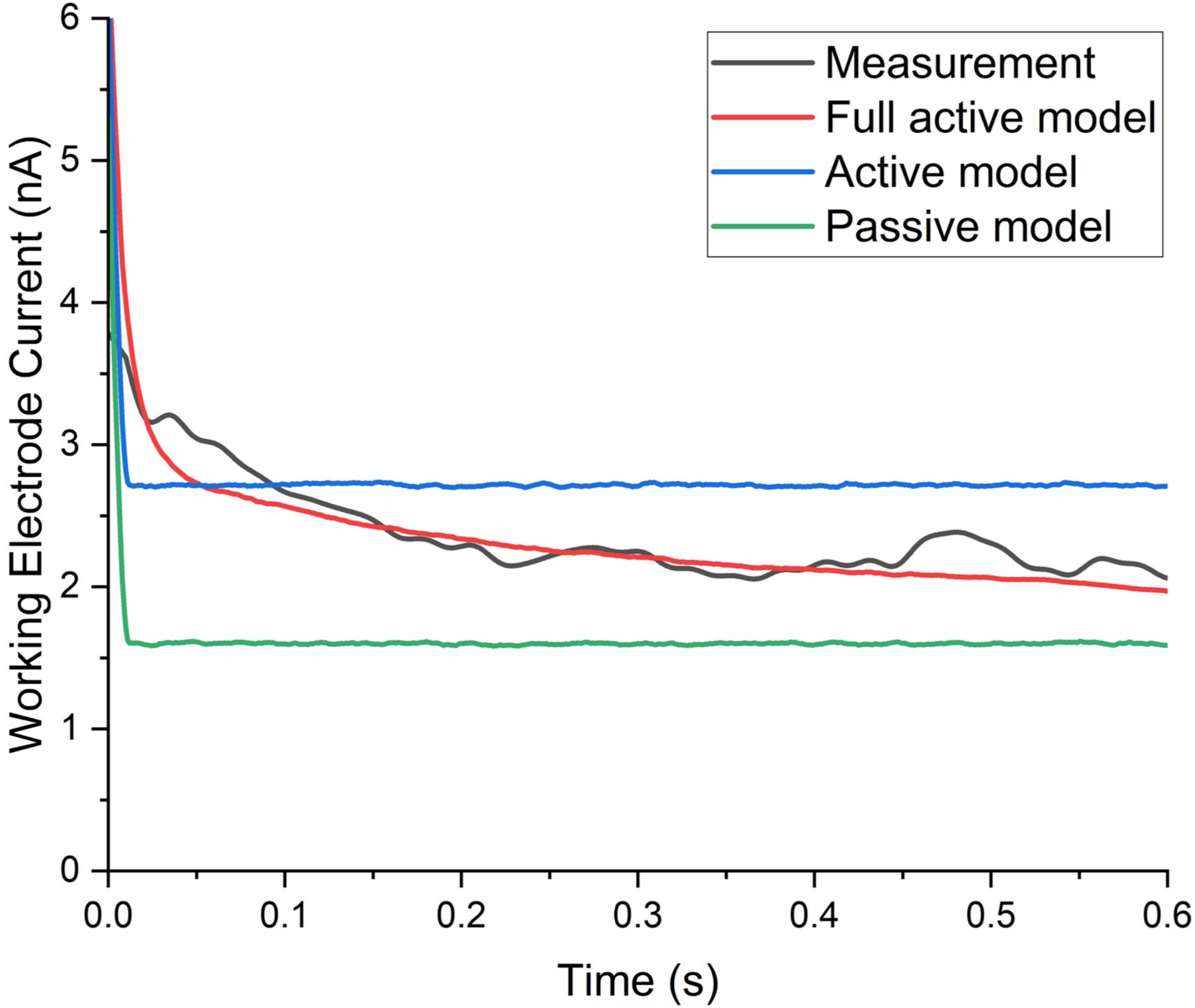
Phase in Frequency of Bio/CMOS interfaces in micron sizes



Risk of Saturation in Chemical Sensors



Transient Behaviors in Chemical Sensor



The Right Equivalent Circuit